Lecture 5: Graphs.

Graphs!

Lecture 5: Graphs.

Graphs!
Euler

Lecture 5: Graphs.

Graphs!
- Euler
- Definitions: model.

Lecture 5: Graphs.

Graphs!
- Euler
- Definitions: model.
- Fact!

Lecture 5: Graphs.

Graphs!

- Euler
- Definitions: model.
- Fact!
- Euler Again!!

Lecture 5: Graphs.

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- Fact!
- Euler Again!!

Lecture 5: Graphs.

Graphs!
- Euler
- Definitions: model.
- Fact!
- Euler Again!!

Planar graphs.

Lecture 5: Graphs.

- Graphs!
  - Euler
  - Definitions: model.
  - Fact!
  - Euler Again!!
- Planar graphs.
  - Euler Again!!!!

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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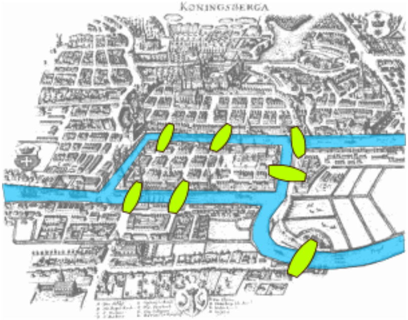

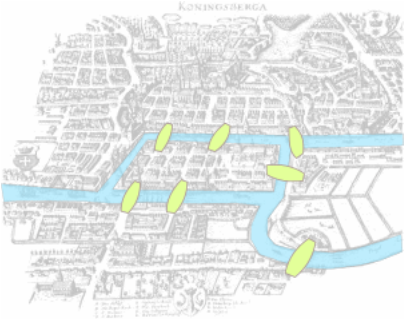

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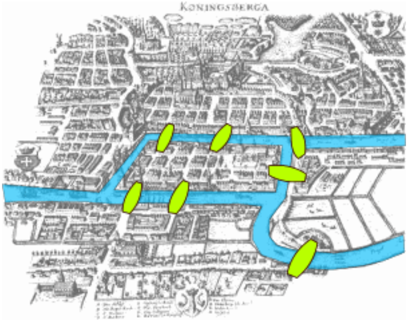

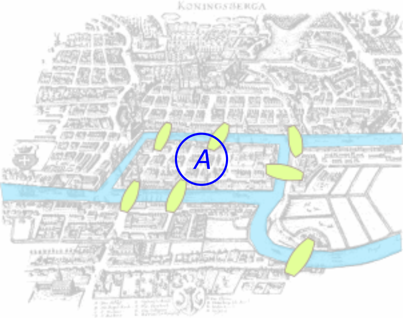

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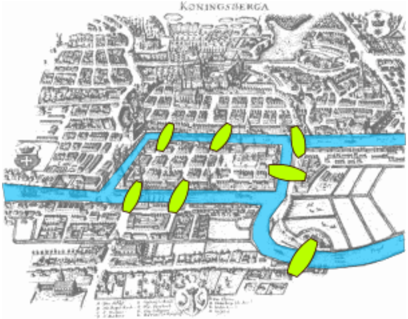

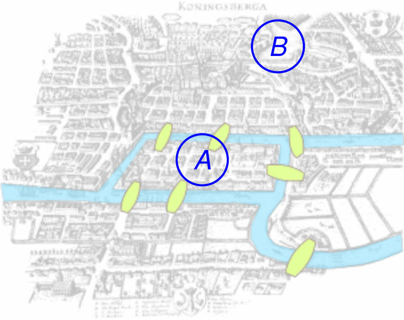

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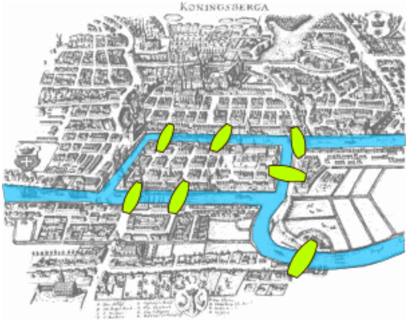

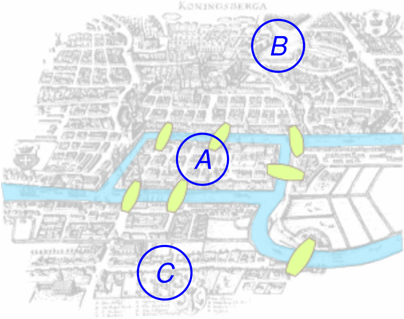

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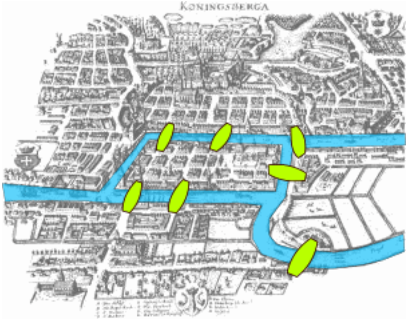

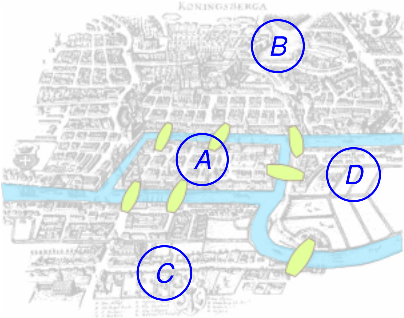

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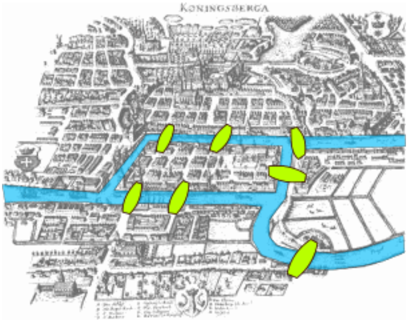

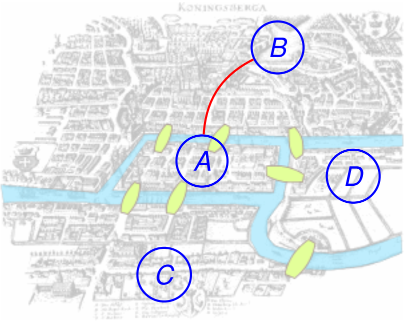

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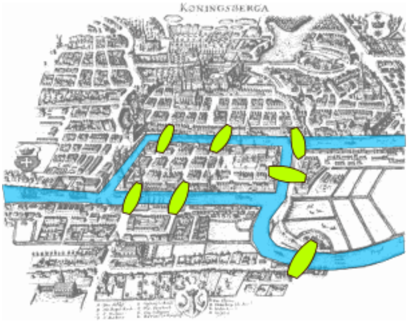

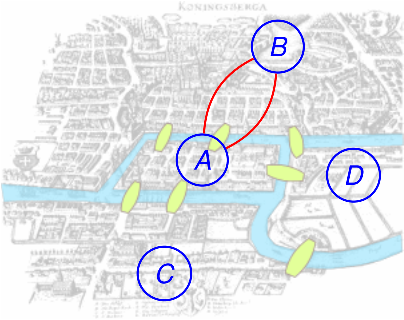

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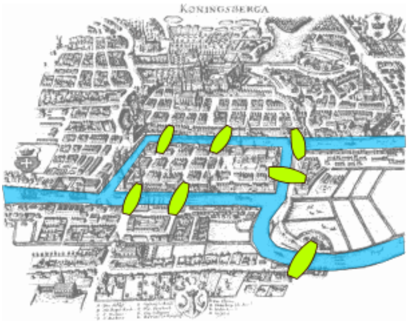

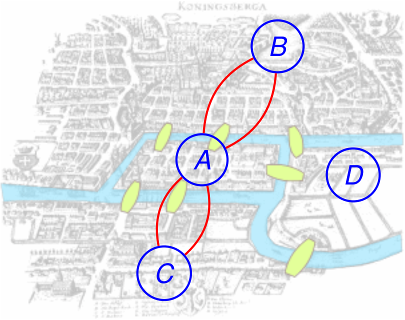

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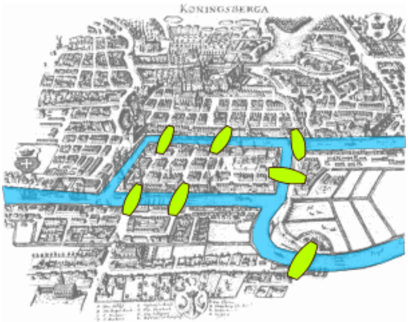

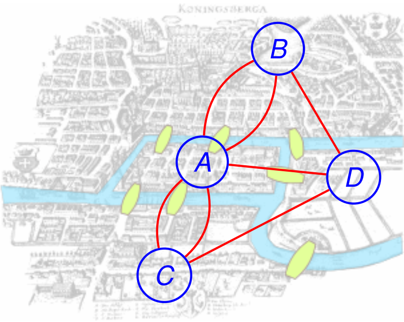

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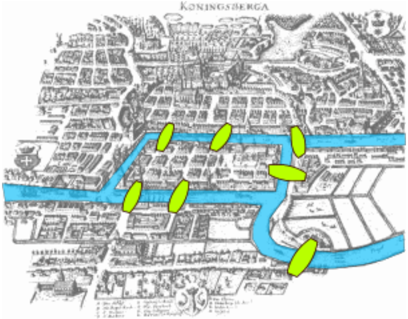

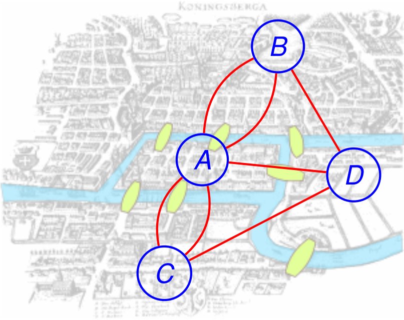

Can you draw a tour in the graph where you visit each edge once?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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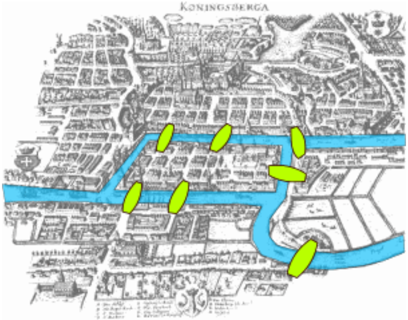

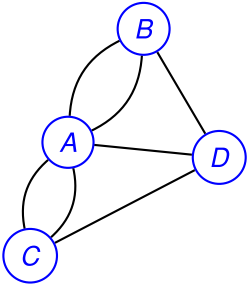

Can you draw a tour in the graph where you visit each edge once?
Yes?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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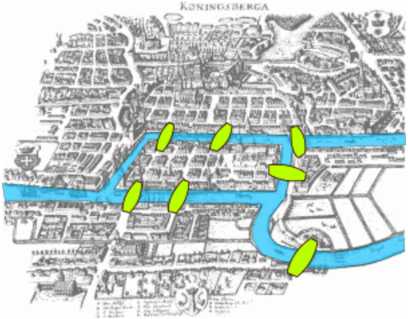

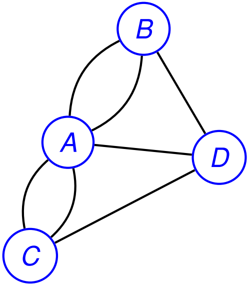

Can you draw a tour in the graph where you visit each edge once?
Yes? No?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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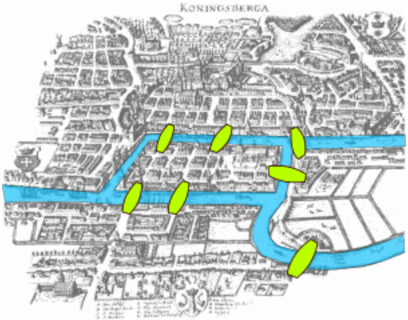
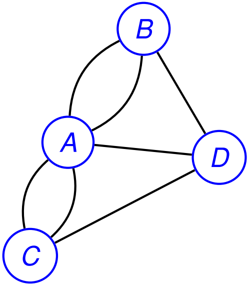
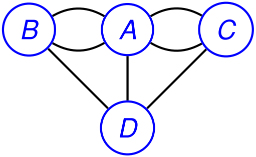




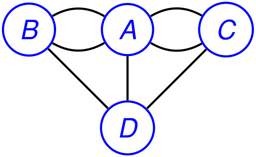
Can you draw a tour in the graph where you visit each edge once?
Yes? No?
We will see!

Graphs: formally.



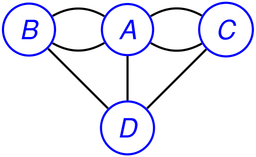
Graph:

Graphs: formally.



Graph: $G = (V, E)$.

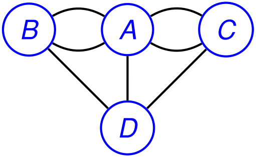
Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

Graphs: formally.

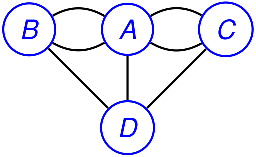


Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

Graphs: formally.



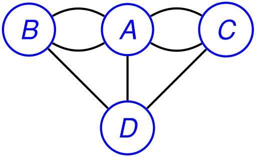
Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ -

Graphs: formally.



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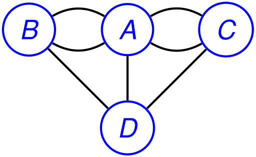


Graph: $G = (V, E)$.

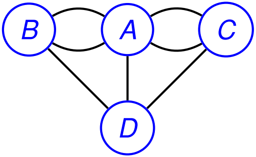
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}$

Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

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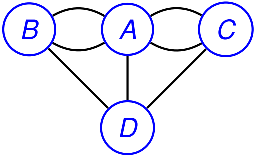
$\{\{A, B\}, \{A, B\}$

Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

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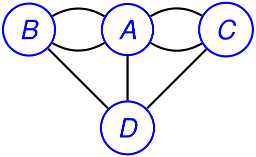
$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\},$

Graphs: formally.



Graph: $G = (V, E)$.

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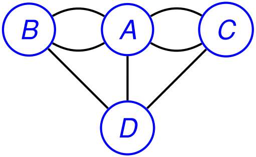
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{B, C\}, \{B, D\}, \{B, D\}, \{C, D\}\}$.

Graphs: formally.



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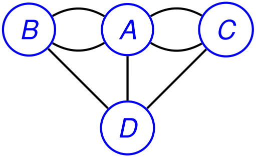
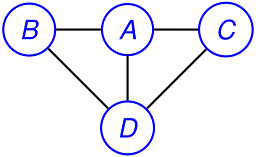
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{B, C\}, \{B, D\}, \{B, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

Graphs: formally.





Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A,B\},\{A,B\},\{A,C\},\{B,C\},\{B,D\},\{B,D\},\{C,D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

Graphs: formally.

Graph: $G = (V, E)$.

V - set of vertices.

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$E \subseteq V \times V$ - set of edges.

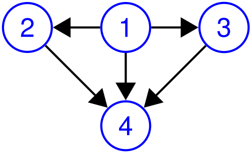
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{B, C\}, \{B, D\}, \{B, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

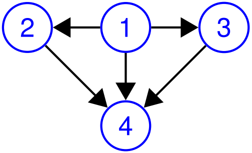
Multigraph above.

Directed Graphs



$G = (V, E)$.

Directed Graphs



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V - set of vertices.

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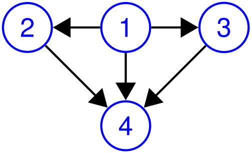
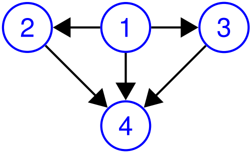


$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

Directed Graphs



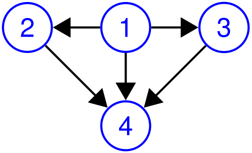
$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

Directed Graphs



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V - set of vertices.

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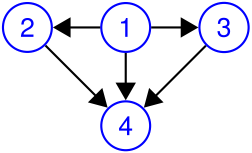
E ordered pairs of vertices.

$\{(1, 2),$

Directed Graphs



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E ordered pairs of vertices.

$\{(1, 2), (1, 3),$

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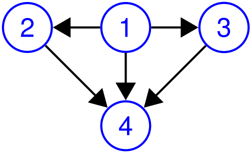

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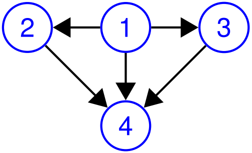
E ordered pairs of vertices.
$\{(1, 2), (1, 3), (1, 4),$

Directed Graphs



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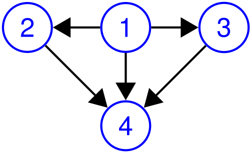
E ordered pairs of vertices.

$\{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\}$

Directed Graphs



$G = (V, E)$.

V - set of vertices.

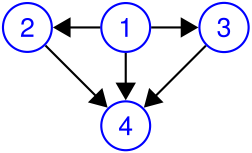
$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.

Directed Graphs



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V - set of vertices.
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E ordered pairs of vertices.
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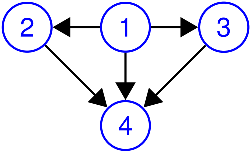
One way streets.
Tournament:

Directed Graphs



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V - set of vertices.

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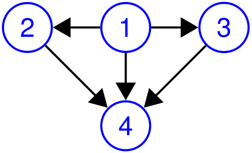
E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2,

Directed Graphs



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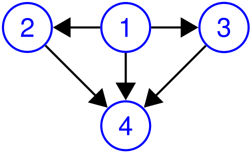
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence:

Directed Graphs



$G = (V, E)$.

V - set of vertices.

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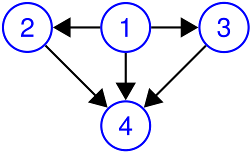
E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2,

Directed Graphs



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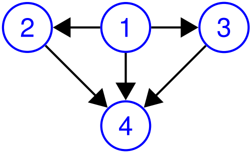
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Directed Graphs



$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

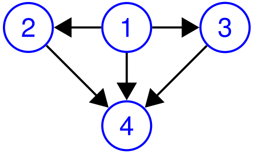
E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network:

Directed Graphs



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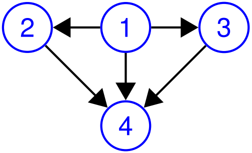
Directed Graphs



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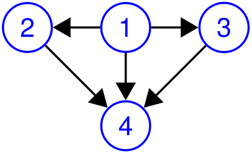
Social Network: Directed? Undirected?

Directed Graphs



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Tournament: 1 beats 2, ...
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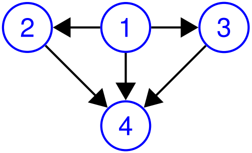
Social Network: Directed? Undirected?
Friends.

Directed Graphs



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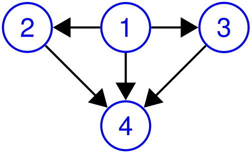
Social Network: Directed? Undirected?
Friends. Undirected.

Directed Graphs



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One way streets.
Tournament: 1 beats 2, ...
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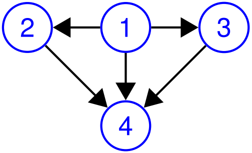
Social Network: Directed? Undirected?
Friends. Undirected.
Likes.

Directed Graphs



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One way streets.
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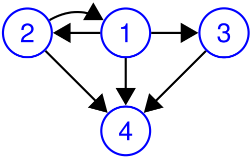
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes. Directed.

Directed Graphs



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One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes. Directed.
Sometimes both ways!

Graph Concepts and Definitions.

Graph: $G = (V, E)$

Graph Concepts and Definitions.

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neighbors, adjacent, degree, incident, in-degree, out-degree

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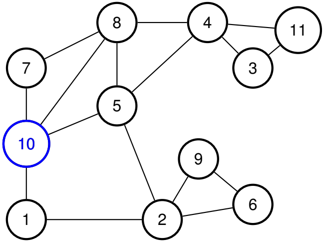
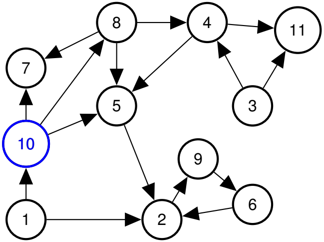


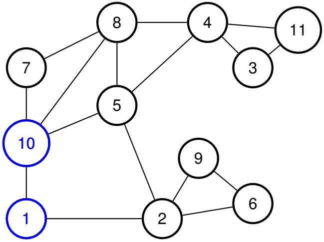


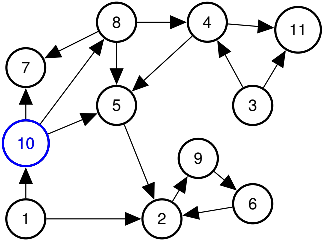
Neighbors of 10?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



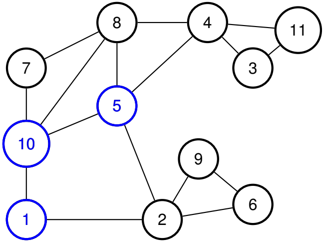


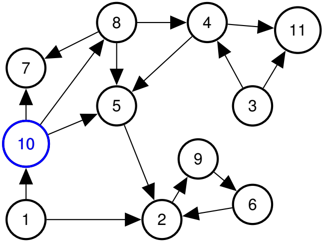
Neighbors of 10? 1,

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree





Neighbors of 10? 1, 5,

Graph Concepts and Definitions.

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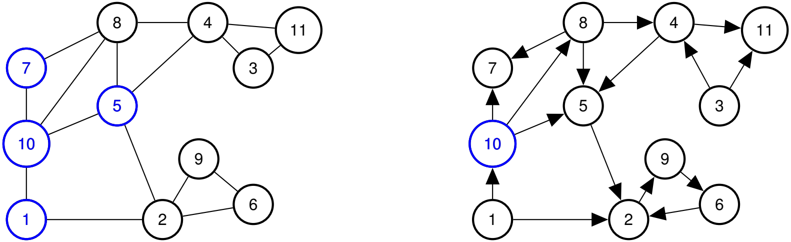
neighbors, adjacent, degree, incident, in-degree, out-degree



Neighbors of 10? 1, 5, 7,

Graph Concepts and Definitions.

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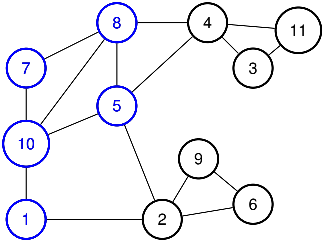

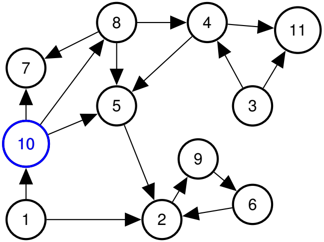

Neighbors of 10?   1, 5, 7, 8.

Graph Concepts and Definitions.

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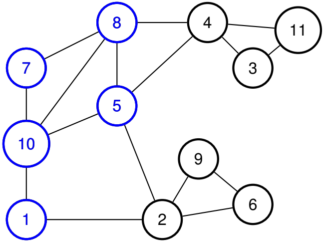

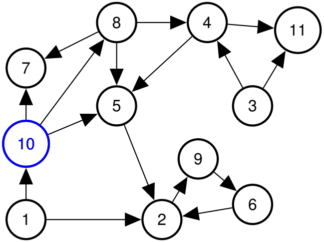

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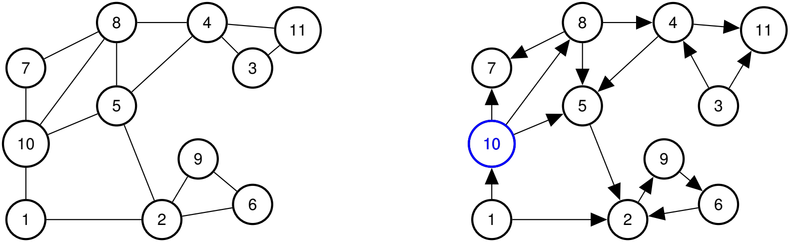

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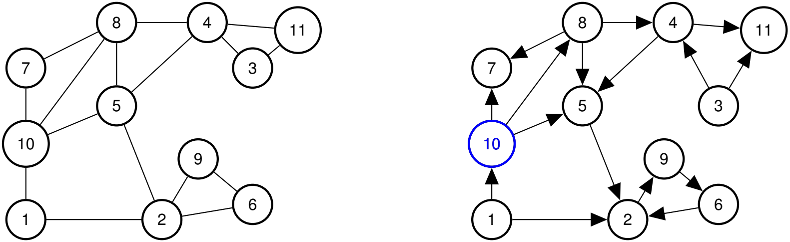

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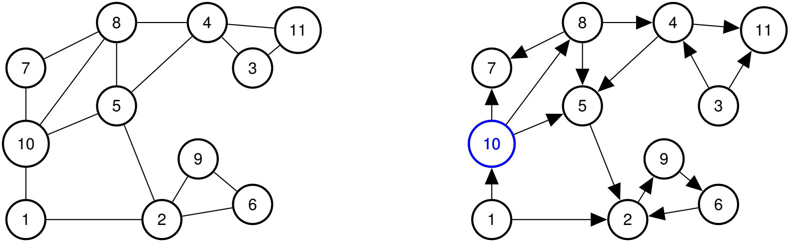

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Degree of vertex 1?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

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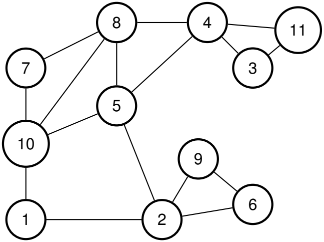

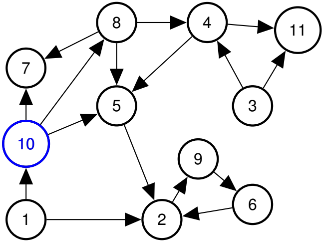

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Degree of vertex 1? 2

Graph Concepts and Definitions.

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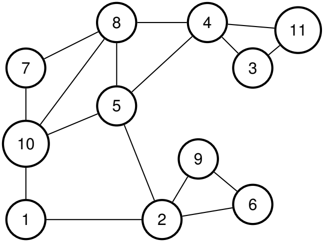
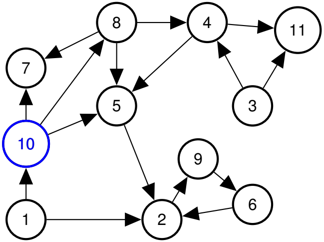
neighbors, adjacent, degree, incident, in-degree, out-degree





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Graph Concepts and Definitions.

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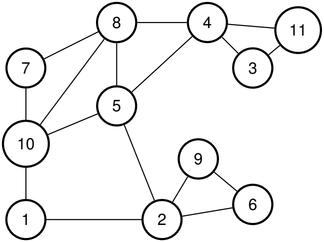

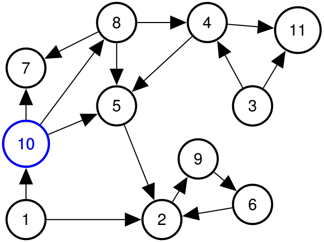

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Equals number of neighbors in simple graph.

Graph Concepts and Definitions.

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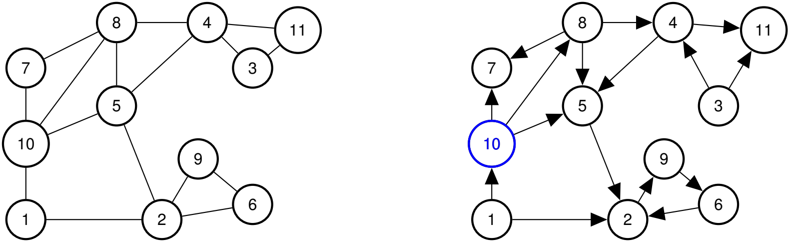
neighbors, adjacent, degree, incident, in-degree, out-degree



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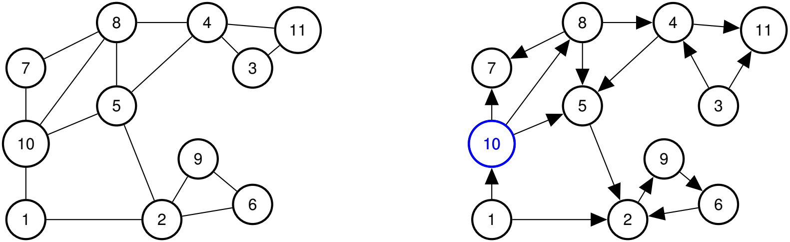

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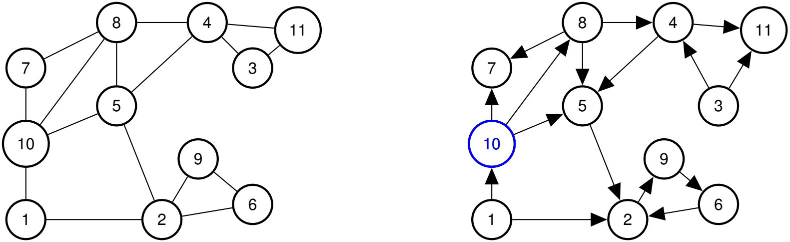
Directed graph?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



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Directed graph?

In-degree of 10?

Graph Concepts and Definitions.

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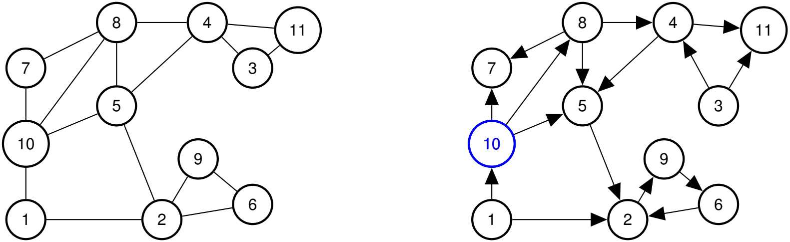
neighbors, adjacent, degree, incident, in-degree, out-degree



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Directed graph?

In-degree of 10? 1

Graph Concepts and Definitions.

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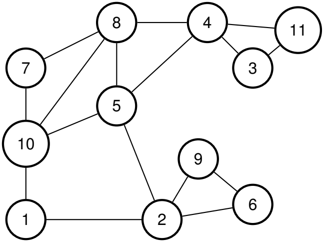

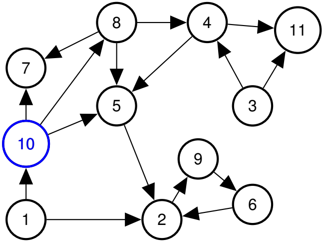

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Directed graph?

In-degree of 10? 1 Out-degree of 10?

Graph Concepts and Definitions.

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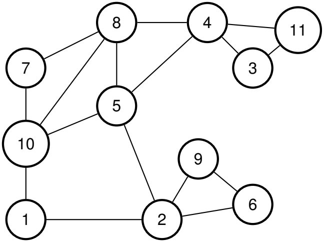

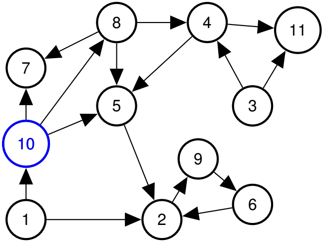

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Degree of vertex 1? 2

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Equals number of neighbors in simple graph.

Directed graph?

In-degree of 10? 1 Out-degree of 10? 3

Graph Concepts and Definitions.

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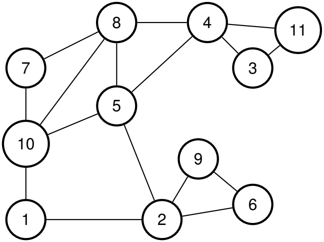
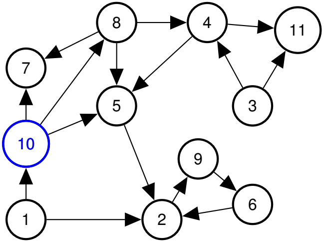
neighbors, adjacent, degree, incident, in-degree, out-degree





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Directed graph?

In-degree of 10? 1   Out-degree of 10? 3

Quick Proof.

The sum of the vertex degrees is equal to

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(A) the total number of vertices, $|V|$.

Quick Proof.

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(B) the total number of edges, $|E|$.

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(C) What?

Quick Proof.

The sum of the vertex degrees is equal to

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(B) the total number of edges, $|E|$.
(C) What?

Not (A)!

Quick Proof.

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(B) the total number of edges, $|E|$.
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Not (A)! Triangle.

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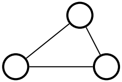

Not (B)!

Quick Proof.

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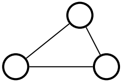

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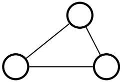

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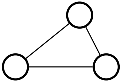

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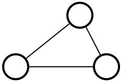


Not (B)! Triangle.

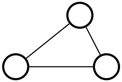
What? For triangle number of edges is 3, the sum of degrees is 6.

Quick Proof.

The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) What?

Not (A)! Triangle.



Not (B)! Triangle.

What? For triangle number of edges is 3, the sum of degrees is 6.

Could it always be...

Quick Proof.

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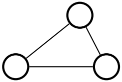

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What? For triangle number of edges is 3, the sum of degrees is 6.

Could it always be...$2|E|$? ..

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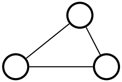

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Could it always be...$2|E|$? ..or $2|V|$?

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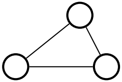

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How many incidences does each edge contribute?

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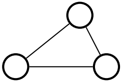

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What? For triangle number of edges is 3, the sum of degrees is 6.

Could it always be...$2|E|$? ..or $2|V|$?

How many incidences does each edge contribute? 2.

Quick Proof.

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(C) What?

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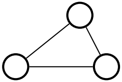

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$2|E|$ incidences are contributed in total!

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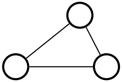

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What is degree v?

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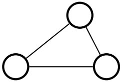

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What is degree v? incidences contributed to v!

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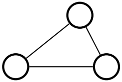

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sum of degrees is total incidences

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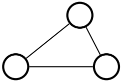

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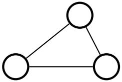

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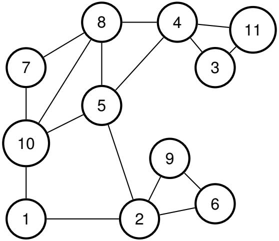
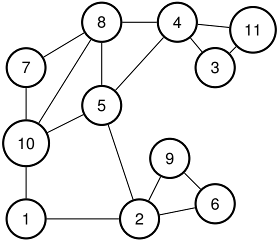
Thm: Sum of vertex degress is $2|E|$.

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Paths, walks, cycles, tour.



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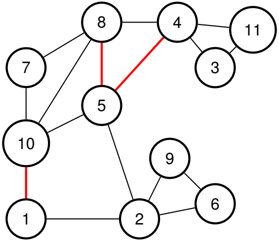
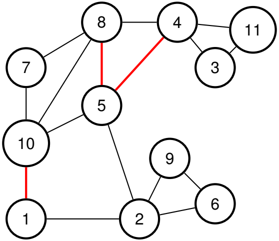


A path in a graph is a sequence of edges.

Path? $\{1,10\}$, $\{8,5\}$, $\{4,5\}$?

Paths, walks, cycles, tour.



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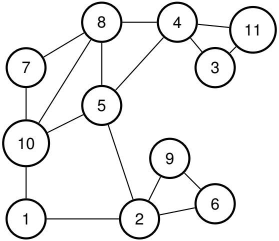

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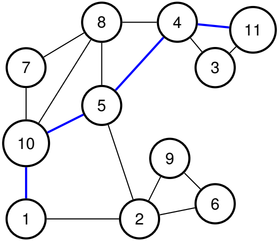

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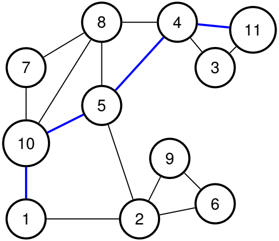
Paths, walks, cycles, tour.



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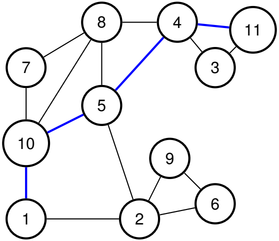

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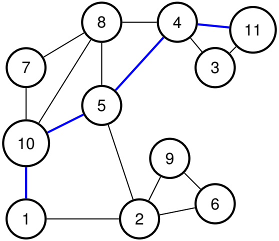
Paths, walks, cycles, tour.



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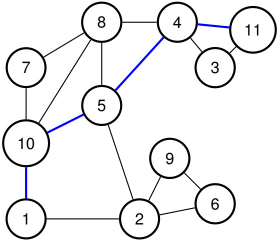
Quick Check!

Paths, walks, cycles, tour.



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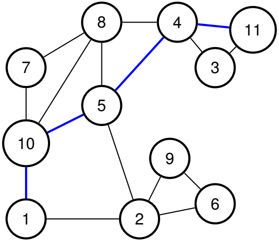
Quick Check! Length of path?

Paths, walks, cycles, tour.



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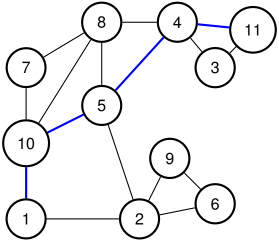
Quick Check! Length of path? k vertices

Paths, walks, cycles, tour.



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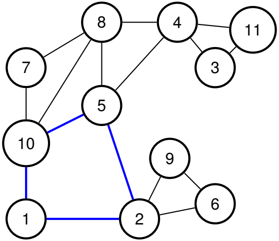
Quick Check! Length of path? k vertices or $k-1$ edges.

Paths, walks, cycles, tour.



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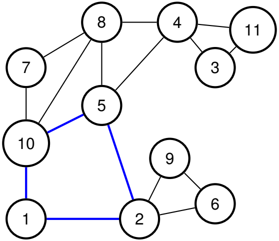
Quick Check! Length of path? k vertices or $k-1$ edges.

Cycle: Path with $v_1 = v_k$.

Paths, walks, cycles, tour.



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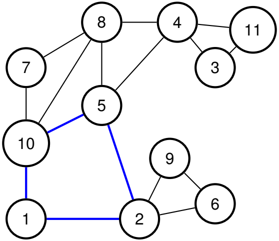
Paths, walks, cycles, tour.



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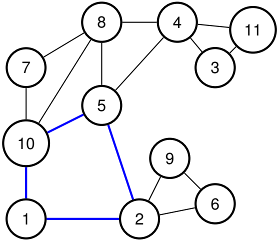
Cycle: Path with $v_1 = v_k$. Length of cycle? $k-1$ vertices and edges!

Paths, walks, cycles, tour.



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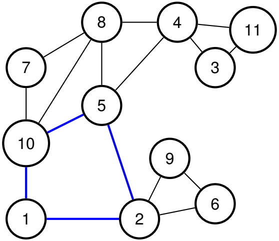
Cycle: Path with $v_1 = v_k$. Length of cycle? $k-1$ vertices and edges!
Path is usually simple.

Paths, walks, cycles, tour.



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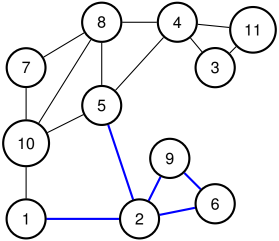
Paths, walks, cycles, tour.



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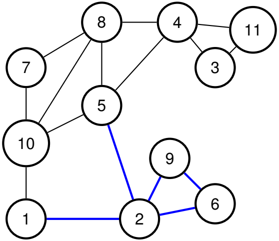
Walk is sequence of edges with possible repeated vertex or edge.

Paths, walks, cycles, tour.



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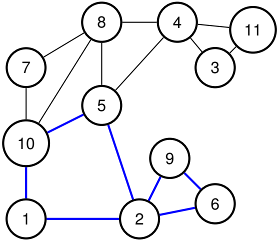

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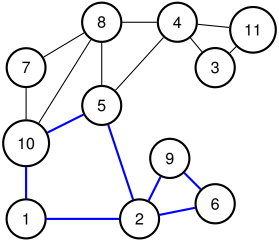
Paths, walks, cycles, tour.



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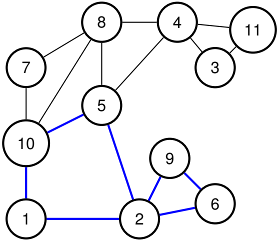

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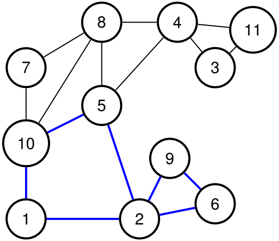
Paths, walks, cycles, tour.



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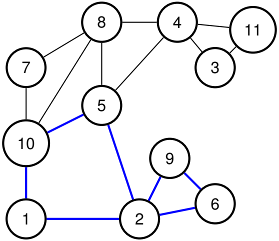
Quick Check!
Path is to Walk as Cycle is to ??

Paths, walks, cycles, tour.



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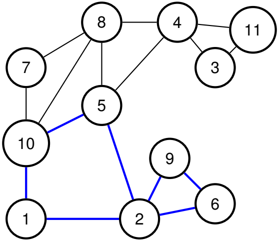
Path is to Walk as Cycle is to ?? Tour!

Paths, walks, cycles, tour.



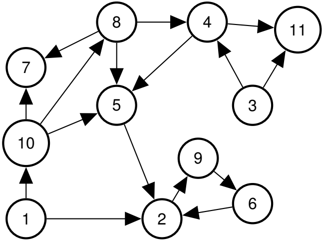
A path in a graph is a sequence of edges.

Path? $\{1,10\}, \{8,5\}, \{4,5\}$? No!
Path? $\{1,10\}, \{10,5\}, \{5,4\}, \{4,11\}$? Yes!

Path: $(v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)$.
Quick Check! Length of path? k vertices or $k-1$ edges.
Cycle: Path with $v_1 = v_k$. Length of cycle? $k-1$ vertices and edges!
Path is usually simple. No repeated vertex!
Walk is sequence of edges with possible repeated vertex or edge.
Tour is walk that starts and ends at the same node.

Quick Check!
Path is to Walk as Cycle is to ?? Tour!

Directed Paths.



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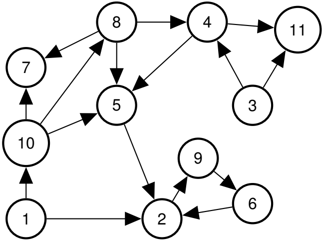

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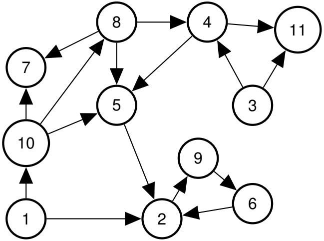

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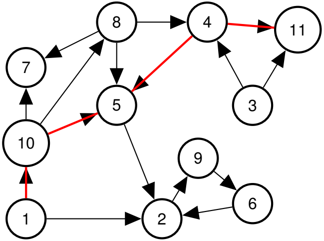

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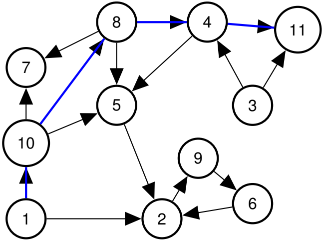

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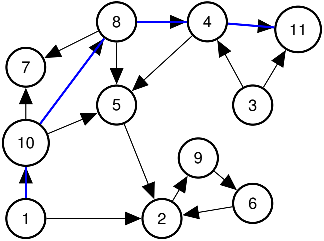

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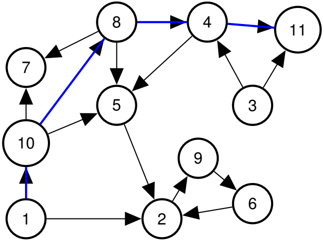

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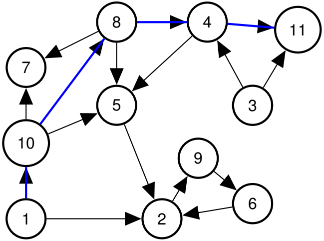

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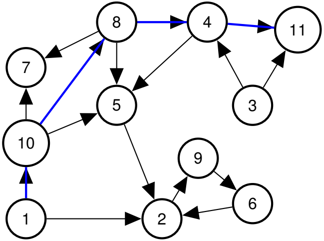

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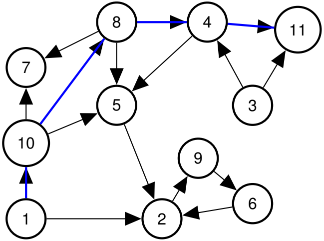

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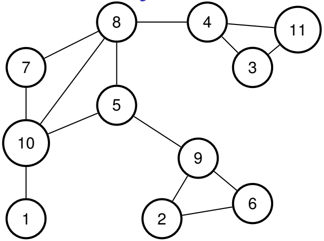
Paths, walks, cycles, tours ... are analagous to undirected now.

Connectivity: undirected graph.



u and *v* are connected if there is a path (or walk) between *u* and *v*.

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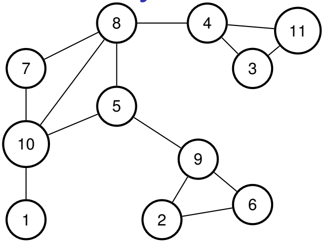

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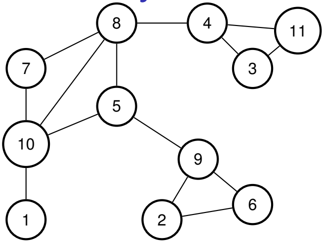

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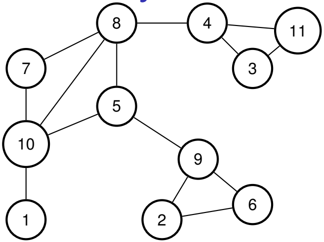

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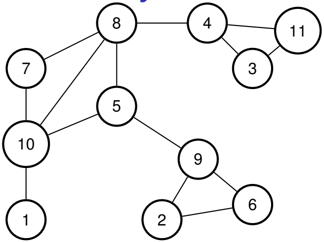

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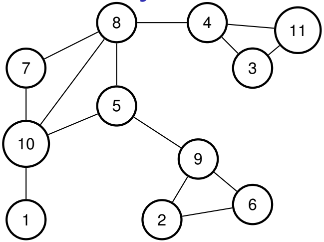

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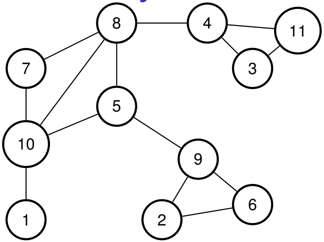

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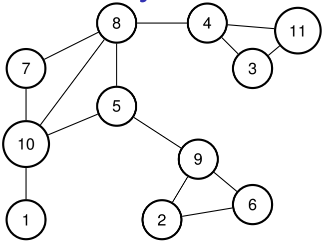

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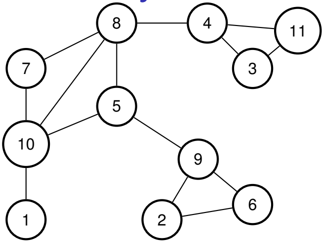

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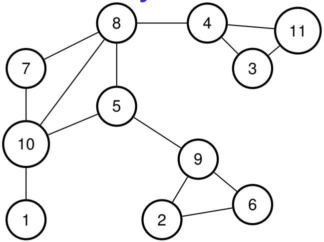

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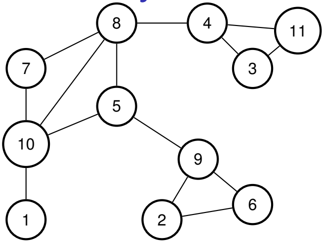

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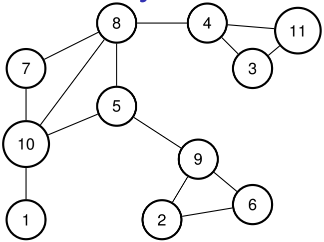

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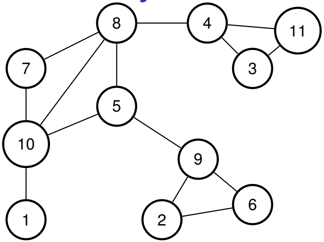

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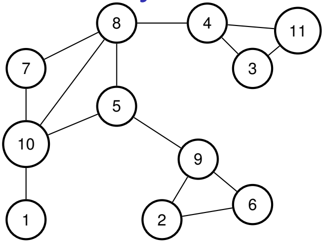

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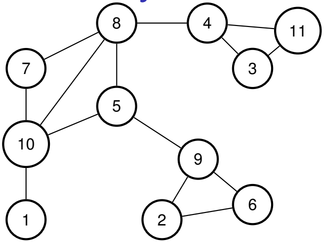

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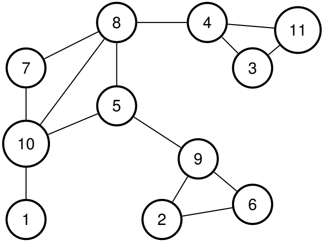
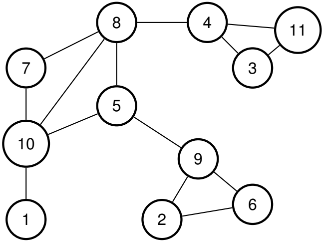


Is graph above connected?

Connected Components



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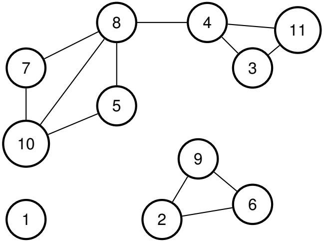
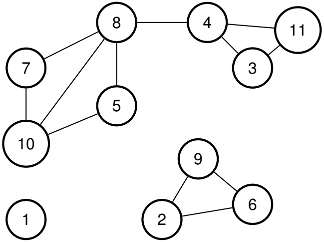


Is graph above connected? Yes!

How about now?

Connected Components



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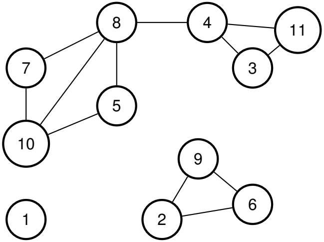

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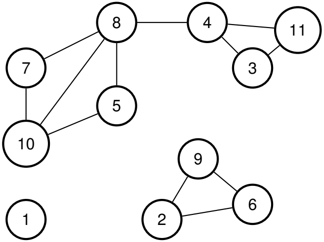

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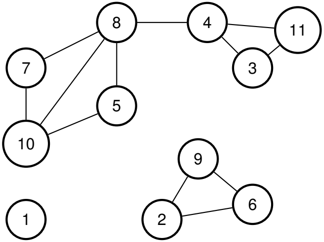
Connected Components? $\{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}$.

Connected Components



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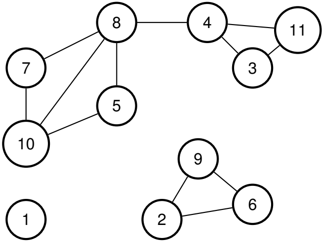
Connected component - maximal set of connected vertices.

Connected Components



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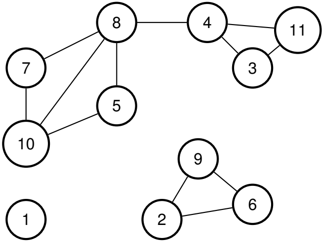
Quick Check: Is $\{10, 7, 5\}$ a connected component?

Connected Components



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Quick Check: Is $\{10, 7, 5\}$ a connected component? No.

Finally..back to Euler!

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Tour enters and leaves vertex v on each visit.

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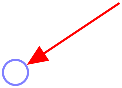

When you enter,

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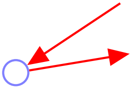

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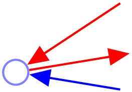

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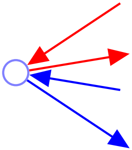

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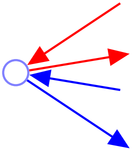

When you enter, you can leave.
For starting node,

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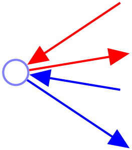
Eulerian Tour is connected so graph is connected.
Tour enters and leaves vertex v on each visit.
Uses two incident edges per visit. Tour uses all incident edges.
Therefore v has even degree. $\square$



When you enter, you can leave.
For starting node, tour leaves first

Finally..back to Euler!

An Eulerian Tour is a tour that visits each edge exactly once.

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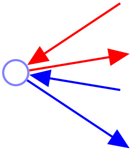

When you enter, you can leave.
For starting node, tour leaves first ....then enters at end.

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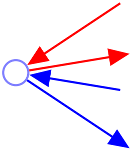

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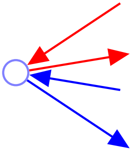

When you enter, you can leave.
For starting node, tour leaves first ....then enters at end.
Not The Hotel California.

Finding a tour!

Proof of if: Even + connected $\Longrightarrow$ Eulerian Tour.
We will give an algorithm.

Finding a tour!

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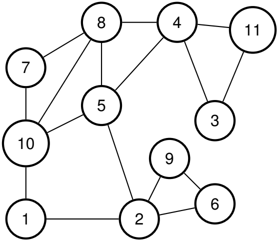

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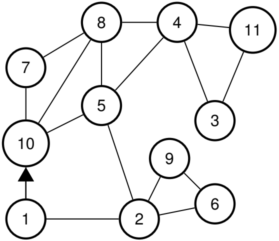

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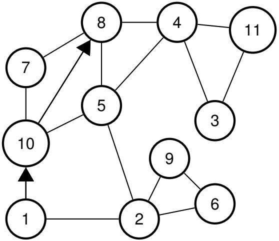

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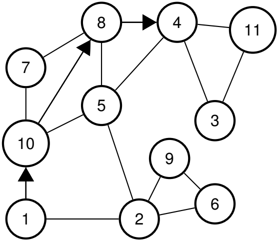

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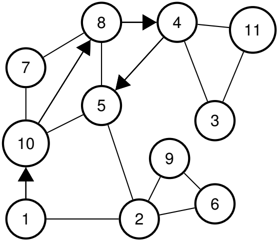

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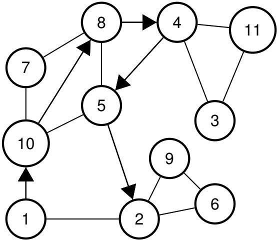

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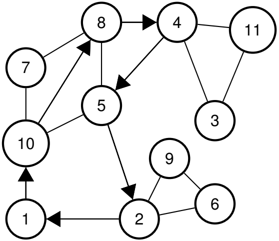

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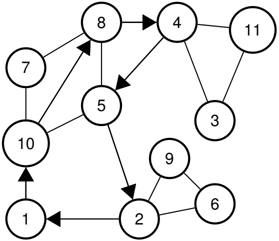

1. Start walk from v (1) on "unused" edges

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2. Remove tour, C.

Finding a tour!

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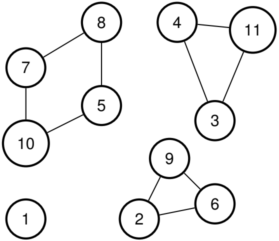

1. Start walk from v (1) on "unused" edges
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Finding a tour!

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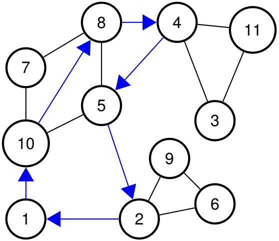

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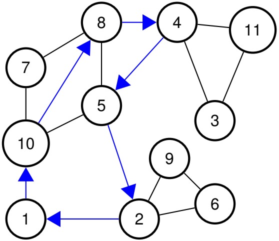

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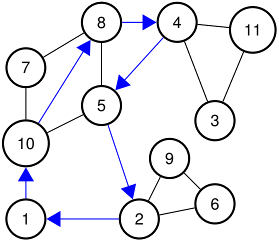

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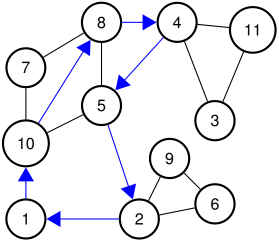

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3. Let $G_1, \ldots, G_k$ be connected components.

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   Why? G was connected.

   Let v_i be (first) node in G_i touched by C.

Finding a tour!

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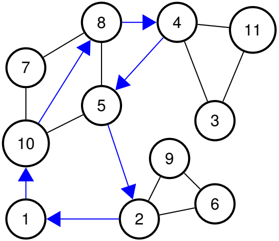

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    Example: $v_1 = 1$,

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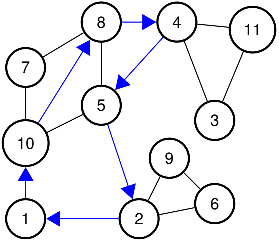

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   Example: $v_1 = 1$, $v_2 = 10$,

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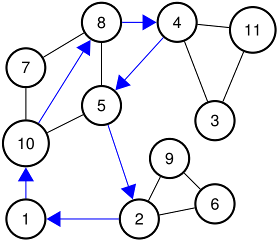

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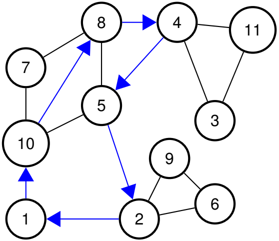

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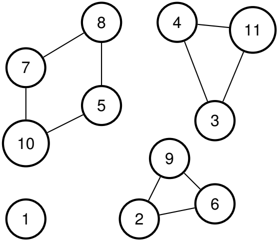

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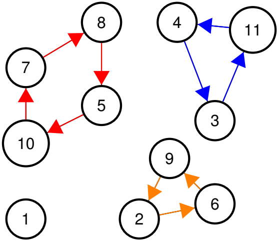

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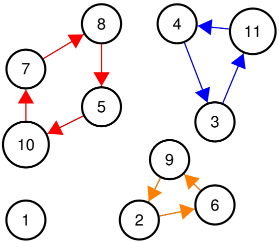

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4. Recurse on $G_1, \ldots, G_k$ starting from v_i
5. Splice together.

Finding a tour!

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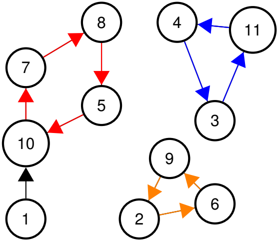

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   1,10

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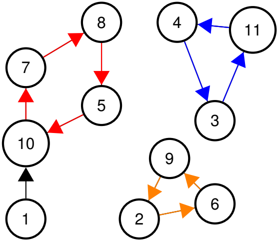

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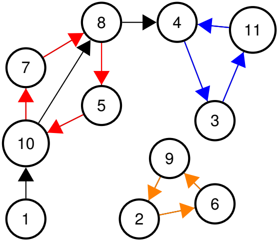

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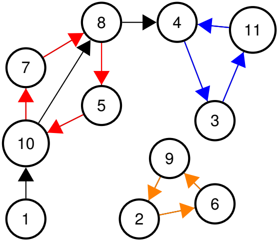

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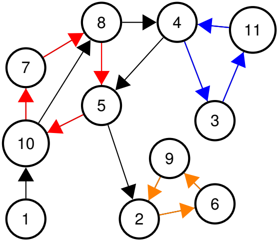

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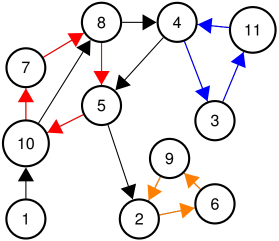

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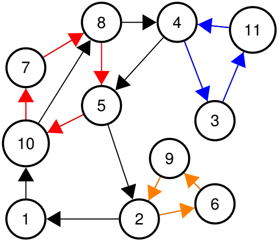

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   1,10,7,8,5,10 ,8,4,3,11,4 5,2,6,9,2 and to 1!

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

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Claim: Do get back to v!

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Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v.

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

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Proof of Claim: Even degree. If enter, can leave except for v. $\square$

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2. Remove cycle, C, from G.

Recursive/Inductive Algorithm.

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Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.
Resulting graph may be disconnected. (Removed edges!)

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.
Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.

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Why is there a v_i in C?

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G was connected $\implies$

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Claim: Each vertex in each G_i has even degree

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Claim: Each vertex in each G_i has even degree and is connected.

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Visits every edge once:
Visits edges in C

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Visits edges in C exactly once.

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Visits every edge once:
Visits edges in C exactly once.
By induction for all edges in each G_i.

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Break time!

Well admin time!

Break time!

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Must choose homework option or test only: soon after recieving hw 1 scores.

Break time!

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Must choose homework option or test only: soon after recieving hw 1 scores.

Test Option: don't have to do homework.

Break time!

Well admin time!

Must choose homework option or test only: soon after recieving hw 1 scores.

Test Option: don't have to do homework. Yes!!

Break time!

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Must choose homework option or test only: soon after recieving hw 1 scores.

Test Option: don't have to do homework. Yes!!
Should do homework.

Break time!

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Must choose homework option or test only: soon after recieving hw 1 scores.

- Test Option: don't have to do homework. Yes!!
  - Should do homework. No need to write up.

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Homework Option: have to do homework. Bummer!

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The truth:

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The truth: mostly test,

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The truth: mostly test, both options!

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- Variance mostly in exams for A/B range.

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Homework Option: have to do homework. Bummer!

The truth: mostly test, both options!
Variance mostly in exams for A/B range.
most homework students get near perfect scores on homework.

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How will I do?

Break time!

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How will I do?

Mostly up to you.

Planar graphs.

A graph that can be drawn in the plane without edge crossings.

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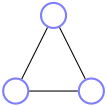

Planar?

Planar graphs.

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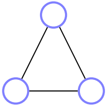

Planar? Yes for Triangle.

Planar graphs.

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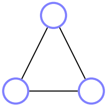
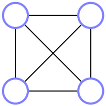
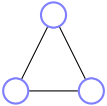
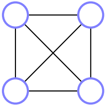
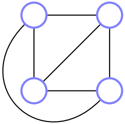
 

Planar? Yes for Triangle.
Four node complete?

Planar graphs.

A graph that can be drawn in the plane without edge crossings.

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Planar graphs.

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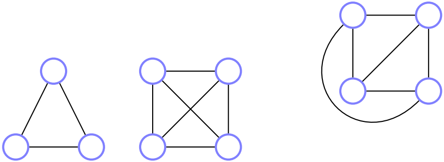

Planar? Yes for Triangle.

Four node complete? Yes.

Note: Complete means every possible edge is present, also clique.

Planar graphs.

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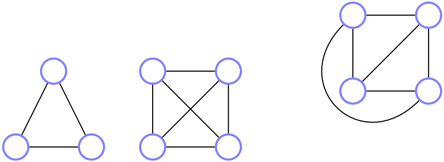

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Five node complete or K_5?

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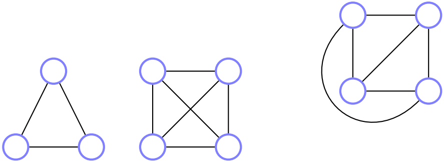

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Five node complete or K_5? No!

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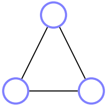
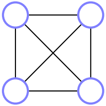
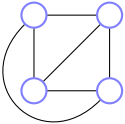

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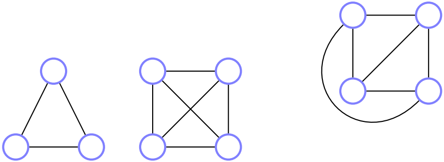

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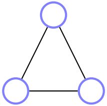 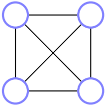 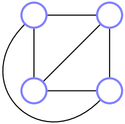

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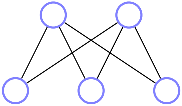

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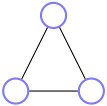

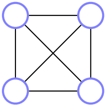

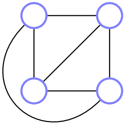

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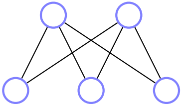
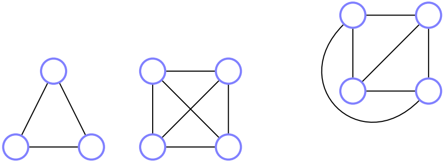


Two to three nodes, bipartite?

Planar graphs.

A graph that can be drawn in the plane without edge crossings.



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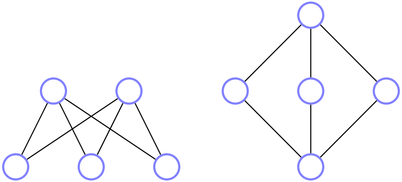

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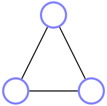
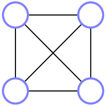
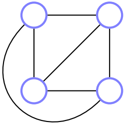
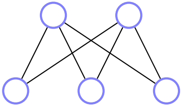
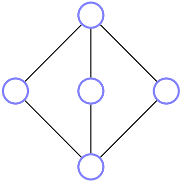
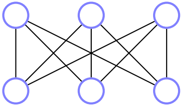




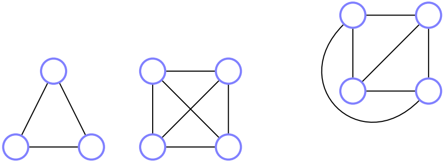
Planar? Yes for Triangle.
Four node complete? Yes.
Note: Complete means every possible edge is present, also clique.
Five node complete or K_5? No! Why? Later.

Two to three nodes, bipartite? Yes.
Three to three nodes, complete/bipartite or $K_{3,3}$.

Planar graphs.

A graph that can be drawn in the plane without edge crossings.



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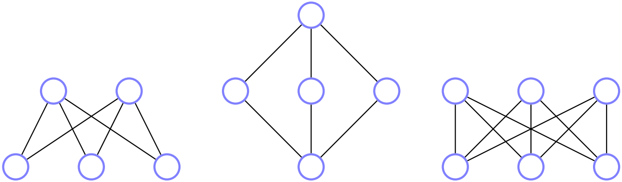

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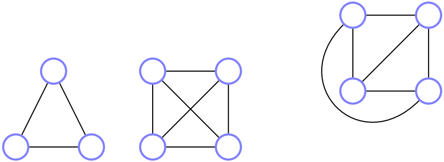

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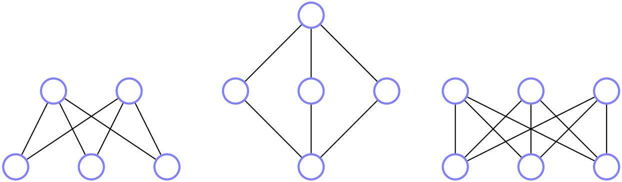

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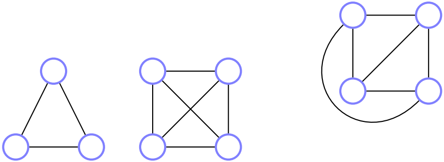

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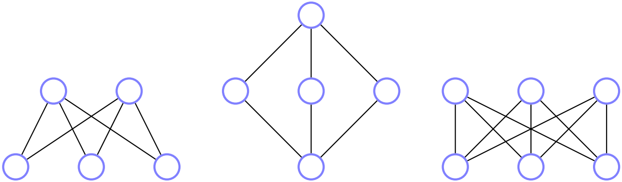

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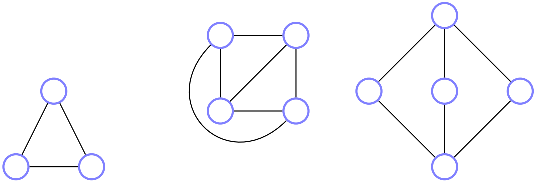
Euler's Formula.



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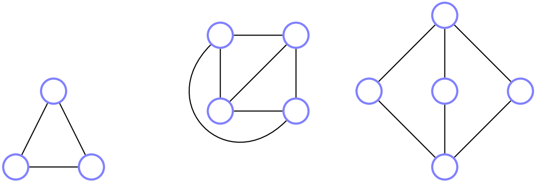
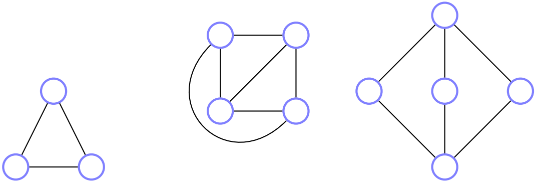


Faces: connected regions of the plane.

Euler's Formula.



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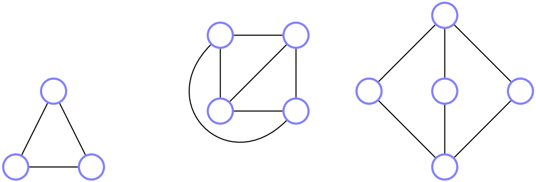
Euler's Formula.

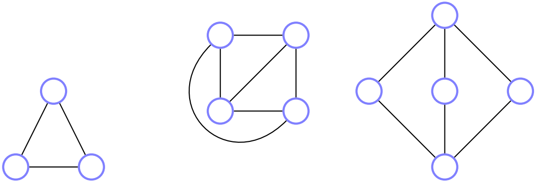


Faces: connected regions of the plane.

How many faces for

triangle?

Euler's Formula.



Faces: connected regions of the plane.

How many faces for
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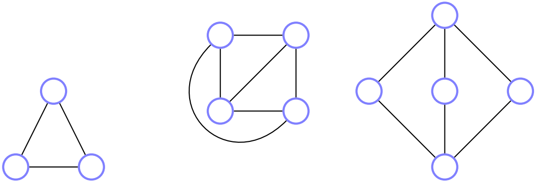
Euler's Formula.



Faces: connected regions of the plane.

How many faces for

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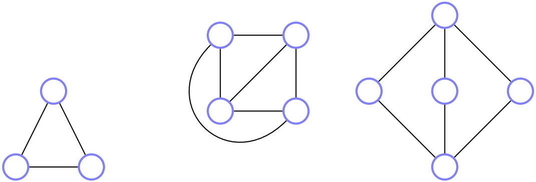

Faces: connected regions of the plane.

How many faces for

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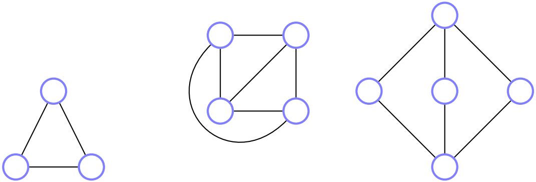
Euler's Formula.



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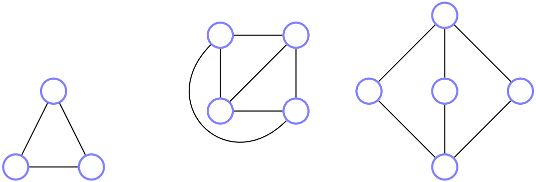

Faces: connected regions of the plane.

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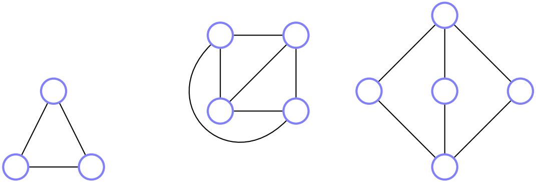
Euler's Formula.



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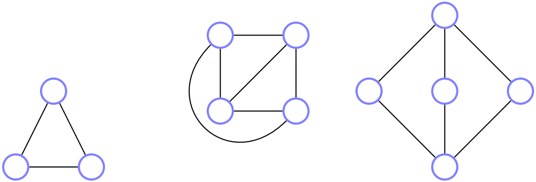

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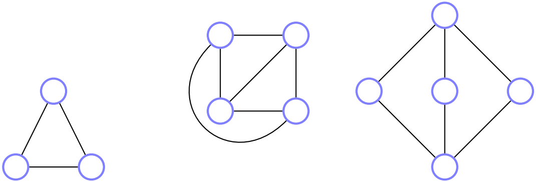

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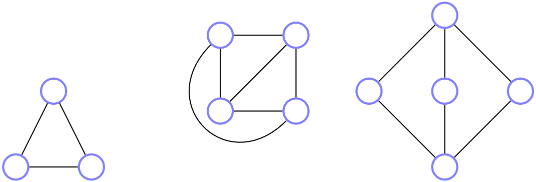

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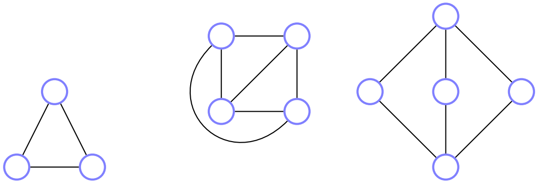

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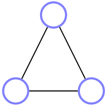 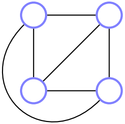 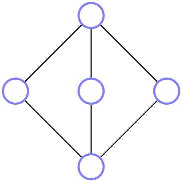

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Triangle: $3 + 2 = 3 + 2$!

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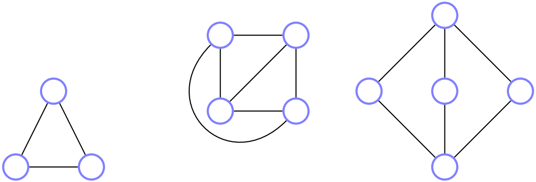

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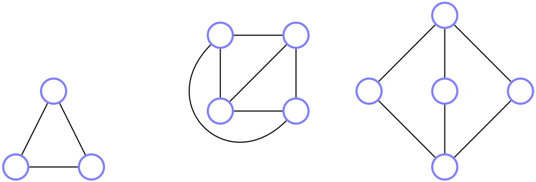

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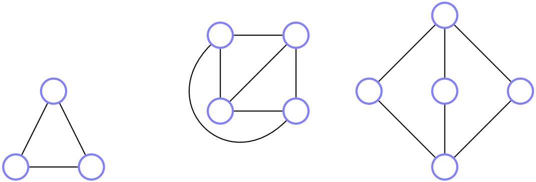

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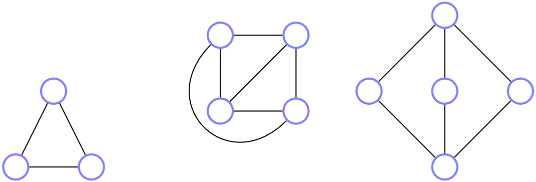

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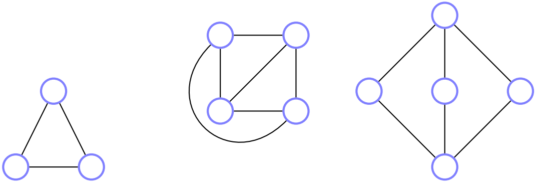

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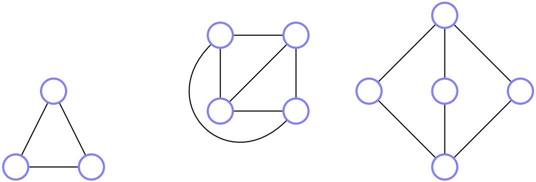

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Examples = 3!

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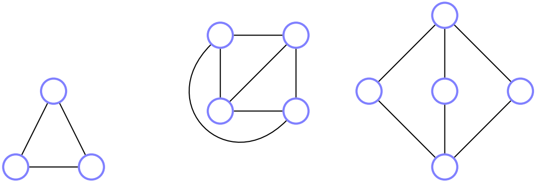

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Examples = 3! Proven!

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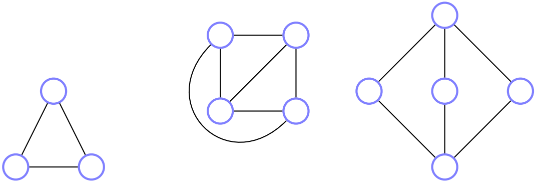

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Euler and Polyhedron.

Greeks knew formula for convex polyhedron.

Euler and Polyhedron.

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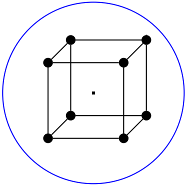
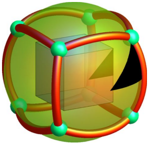
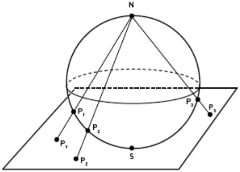
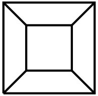


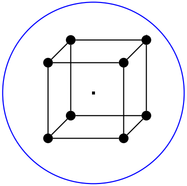


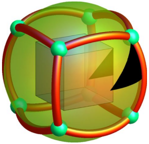


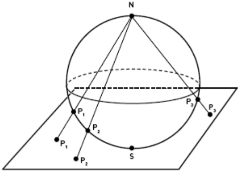


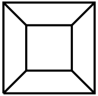
Euler and Polyhedron.

Greeks knew formula for convex polyhedron.









Faces?

Euler and Polyhedron.

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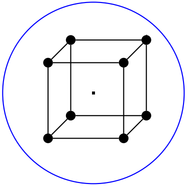

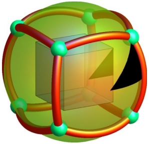

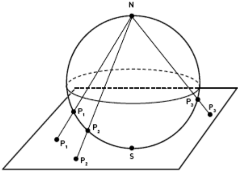

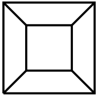

Faces? 6.   Edges?

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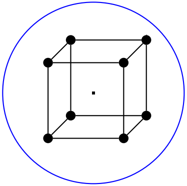

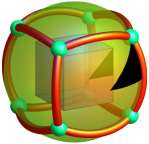

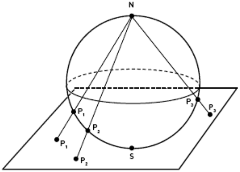

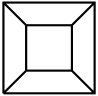

Faces? 6. Edges? 12.

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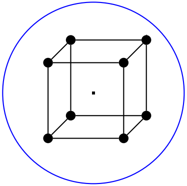

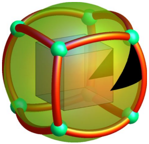

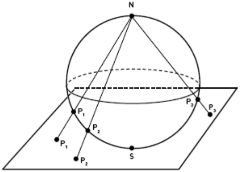

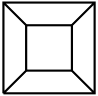

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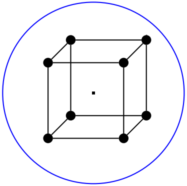

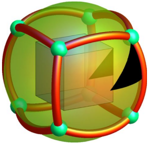

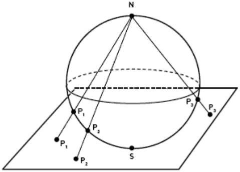

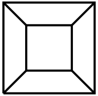

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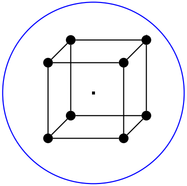

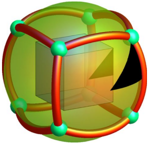

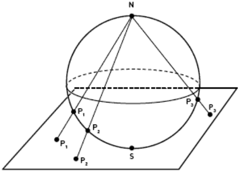

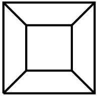

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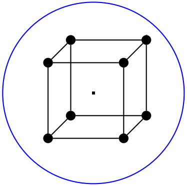
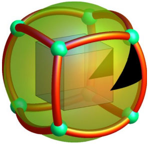
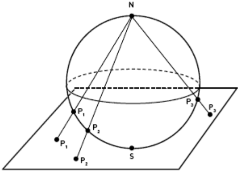

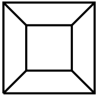

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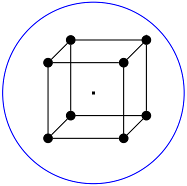
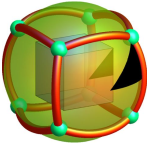
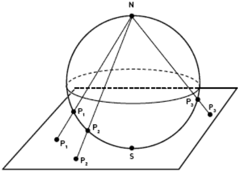

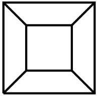

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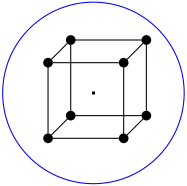

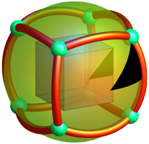

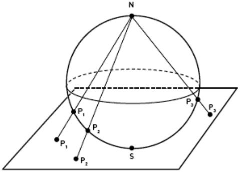

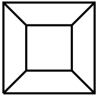

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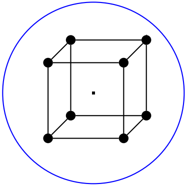

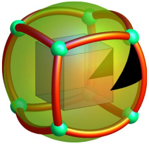

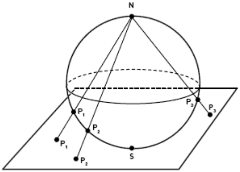

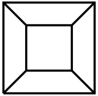

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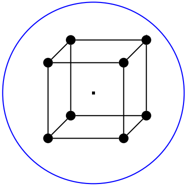
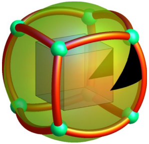
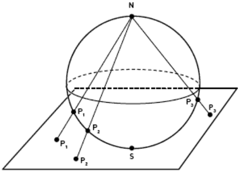

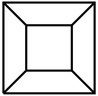

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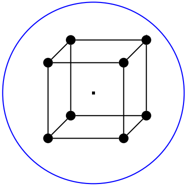

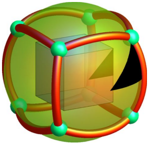

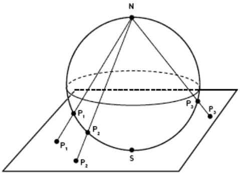

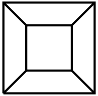

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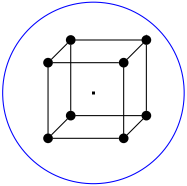

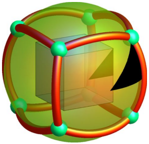

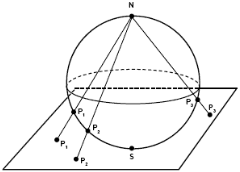

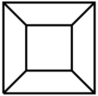

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Convex Polyhedron without holes

Euler and Polyhedron.

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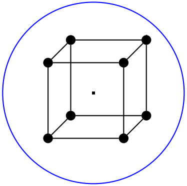
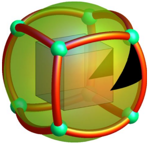
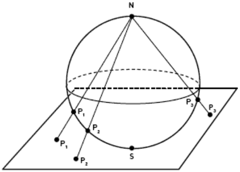

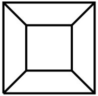

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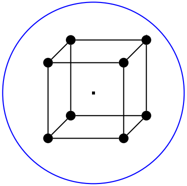

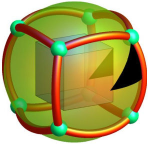

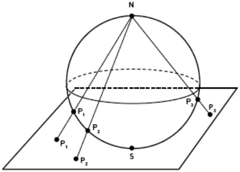

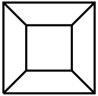

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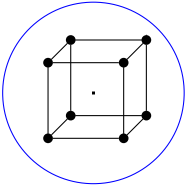

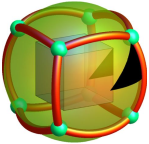

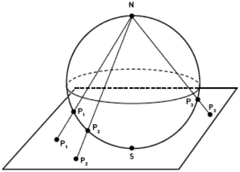

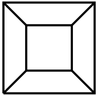

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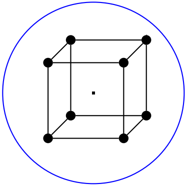
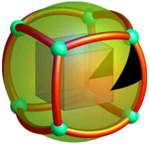
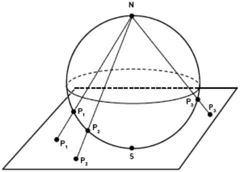
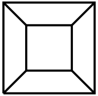
Convex Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.

Euler and Polyhedron.

Greeks knew formula for convex polyhedron.



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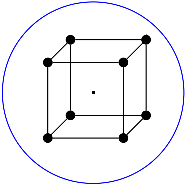
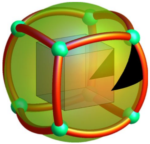
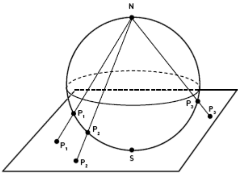
Greeks couldn't prove it. Induction? Remove vertice for polyhedron?

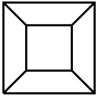
Convex Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.
Project from point inside polytope onto sphere.

Euler and Polyhedron.

Greeks knew formula for convex polyhedron.



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Convex Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.
Project from point inside polytope onto sphere.
Sphere

Euler and Polyhedron.

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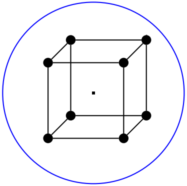
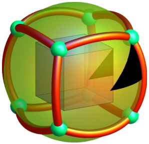
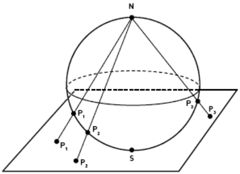

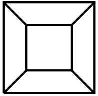

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Convex Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.
Project from point inside polytope onto sphere.
Sphere $\equiv$ Plane!

Euler and Polyhedron.

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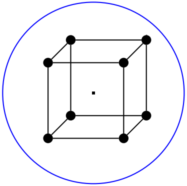

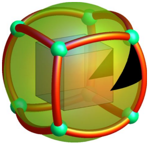

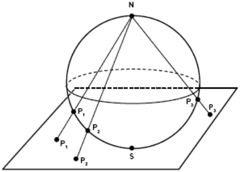

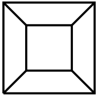

Faces? 6. Edges? 12. Vertices? 8.

Euler: Connected planar graph: $v + f = e + 2$.

$8 + 6 = 12 + 2$.

Greeks couldn't prove it. Induction? Remove vertice for polyhedron?

Convex Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.
Project from point inside polytope onto sphere.
Sphere $\equiv$ Plane! Topologically.

Euler and Polyhedron.

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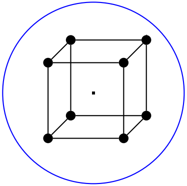

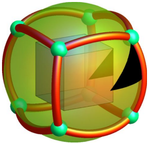

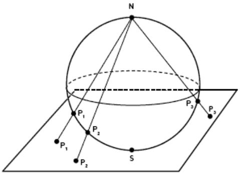

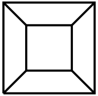

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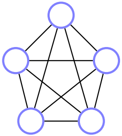
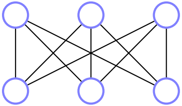
Euler proved formula thousands of years later!

Euler and non-planarity of K_5 and $K_{3,3}$





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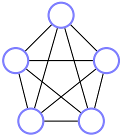

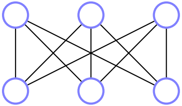

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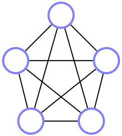

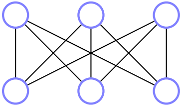

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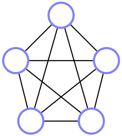

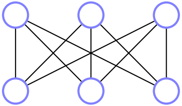

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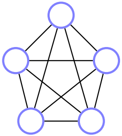

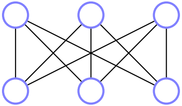

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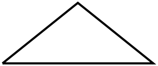

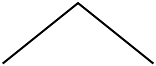

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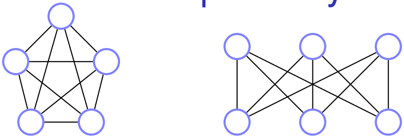

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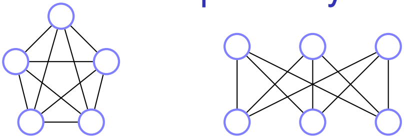

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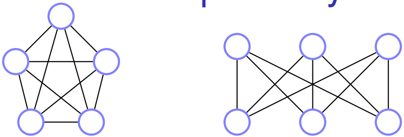

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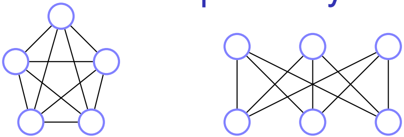

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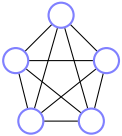

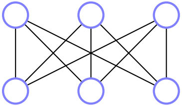

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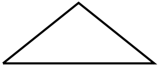

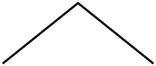

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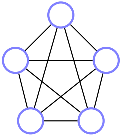
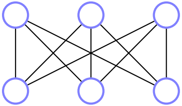

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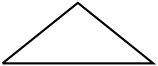
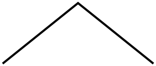

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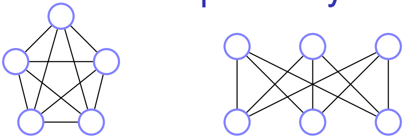

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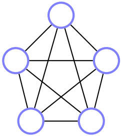

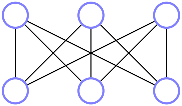

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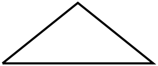

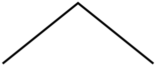

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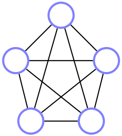

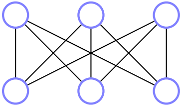

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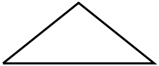

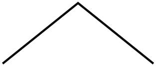

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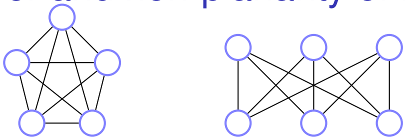

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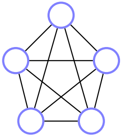

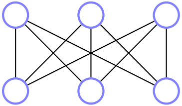

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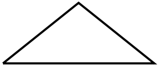

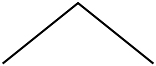

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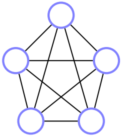

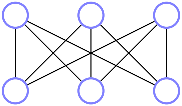

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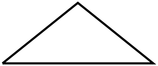

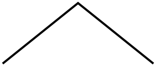

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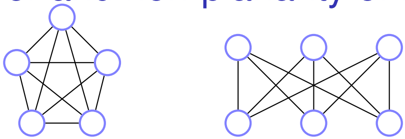

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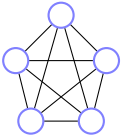

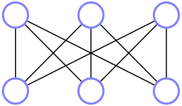

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K_5

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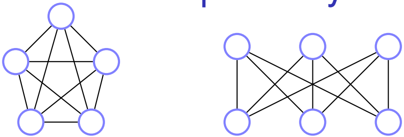

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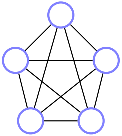
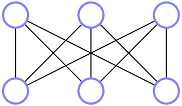
Plug into Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

K_5 Edges?

Euler and non-planarity of K_5 and $K_{3,3}$





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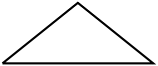

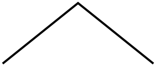

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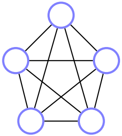

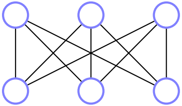

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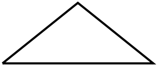

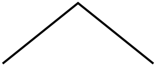

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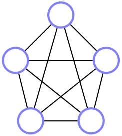

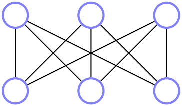

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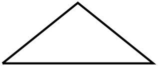

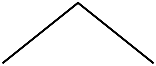

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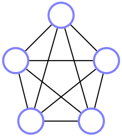

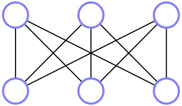

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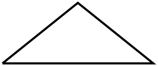

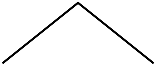

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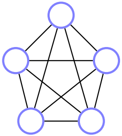

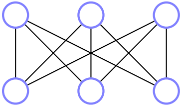

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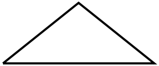

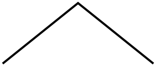

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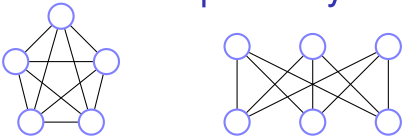
$10 \not\leq 3(5) - 6 = 9$.

Euler and non-planarity of K_5 and $K_{3,3}$



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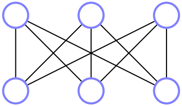
$10 \not\leq 3(5) - 6 = 9$. $\implies$ K_5 is not planar.

Proving non-planarity for $K_{3,3}$



Euler's formula $\implies$ $3f \leq 2e$

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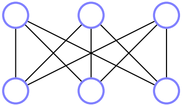

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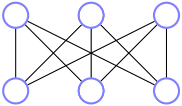

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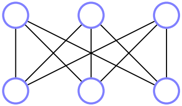

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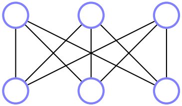

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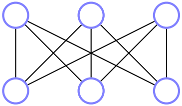

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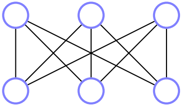

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$$9 \leq 3(6) - 6?$$

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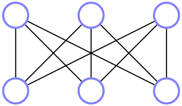

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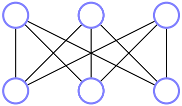

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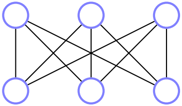

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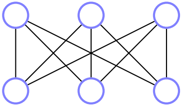

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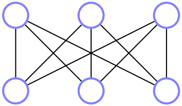

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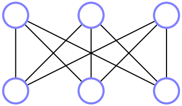

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But no cycles that are triangles. Face is of length ≥ 4.

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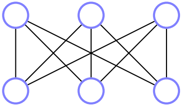

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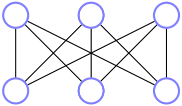

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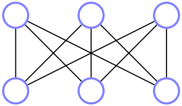

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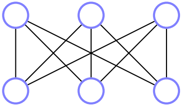

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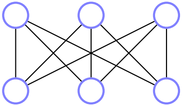

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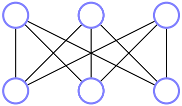

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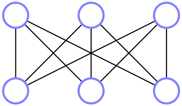

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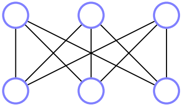

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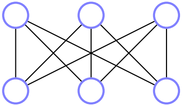

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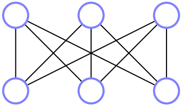

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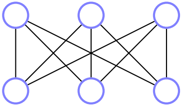
Proving non-planarity for $K_{3,3}$



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$K_{3,3}$? Edges? 9. Vertices. 6.

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Proof doesn't work. Let's fix this.

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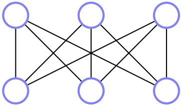

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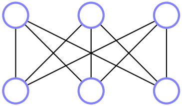

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$9 \not\leq 2(6) - 4$. $\implies$ $K_{3,3}$ is not planar!

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  Basics.

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Non-planarity of K_5 and $K_{3,3}$.

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Next Time: prove Euler's formula.