CS 70 Spring 2019 | Discrete Mathematics and Probability Theory | Satish Rao and Babak Ayazifar | HW 8

Due: March 22, 2019 at 10 PM

Sundry

Before you start your homework, state briefly how you worked on it. Who else did you work with? List names and email addresses. (In case of homework party, you can just describe the group.)

1 Five Coins

We toss a coin five times.

(a) For the first three parts, order matters in the outcome. How many different outcomes are possible?

(b) How many different outcomes are possible with exactly 3 heads?

(c) How many different outcomes are possible with 3 or more heads? Justify your answer with a symmetry argument.

(d) For the next three parts, assume that the coin is unbiased. What is the probability of getting the outcome TTHHH? What is the probability of getting the outcome THHHH?

(e) What's the probability of getting at least one heads?

(f) What's the probability of getting 3 or more heads?

(g) For the next three parts, assume that the coin is biased with probability of heads being $\frac{2}{3}$. What is the probability of getting the outcome TTHHH? What is the probability of getting the outcome THHHH?

(h) What's the probability of getting at least one heads?

(i) What's the probability of getting 3 or more heads?

2 Weathermen

Tom is a weatherman in New York. On days when it snows, Tom correctly predicts the snow 70% of the time. When it doesn't snow, he correctly predicts no snow 95% of the time. In New York, it snows on 10% of all days.

(a) If Tom says that it is going to snow, what is the probability it will actually snow?

(b) What is Tom's overall accuracy?

(c) Tom's friend Jerry is a weatherman in Alaska. Jerry claims that she is a better weatherman than Tom even though her overall accuracy is lower. After looking at their records, you determine that Jerry is indeed better than Tom at predicting snow on snowy days and sun on sunny day. Give an instance of the situation described above.

Hint: what is the weather like in Alaska?

3 Faulty Lightbulbs

Box 1 contains 1000 lightbulbs of which 10% are defective. Box 2 contains 2000 lightbulbs of which 5% are defective.

(a) Suppose a box is given to you at random and you randomly select a lightbulb from the box. If that lightbulb is defective, what is the probability you chose Box 1?

(b) Suppose now that a box is given to you at random and you randomly select two light- bulbs from the box. If both lightbulbs are defective, what is the probability that you chose from Box 1?

4 Solve the Rainbow

Your roommate was having Skittles for lunch and they offer you some. There are five different colors in a bag of Skittles: red, orange, yellow, green, and purple, and there are 20 of each color. You know your roommate is a huge fan of the green Skittles. With probability $1/2$ they ate all of the green ones, with probability $1/4$ they ate half of them, and with probability $1/4$ they only ate 5 green ones.

(a) If you take a Skittle from the bag, what is the probability that it is green?

(b) If you take two Skittles from the bag, what is the probability that at least one is green?

(c) If you take three Skittles from the bag, what is the probability that they are all green?

(d) If all three Skittles you took from the bag are green, what are the probabilities that your roommate had all of the green ones, half of the green ones, or only 5 green ones?

(e) If you take three Skittles from the bag, what is the probability that they are all the same color?

5 Playing Strategically

Bob, Eve and Carol bought new slingshots. Bob is not very accurate hitting his target with probability $1/3$. Eve is better, hitting her target with probability $2/3$. Carol never misses. They decide to play the following game: They take turns shooting each other. For the game to be fair, Bob starts first, then Eve and finally Carol. Any player who gets shot has to leave the game. The last person standing wins the game. What is Bob's best course of action regarding his first shot?

(a) Compute the probability of the event E_1 that Bob wins in a duel against Eve alone, assuming he shoots first. (Hint: Let x be the probability Bob wins in a dual against Eve alone, assuming he fires first. If Bob misses his first shot and then Eve misses her first shot, what is the probability Bob wins in terms of x?)

(b) Compute the probability of the event E_2 that Bob wins in a duel against Eve alone, assuming he shoots second.

(c) Compute the probability of the same events for a duel of Bob against Carol.

(d) Assuming that both Eve and Carol play rationally, conclude that Bob's best course of action is to shoot into the air (i.e., intentionally miss)! (Hint: What happens if Bob misses? What if he doesn't?)

6 Minesweeper

Minesweeper is a game that takes place on a grid of squares. When you click a square, it disappears to reveal either an integer $\in [1,8]$, a mine, or a blank space. If it reveals a mine, you instantly lose. If it reveals a number, that number refers to the number of mines adjacent to that square (including diagonally adjacent). If it reveals a blank space, there were 0 mines adjacent to it.

You are playing on a 8x8 board with 10 mines randomly distributed across the board. In your first move, you click a square near the center of the board.

(a) What is the probability that the square reveals...

   i. a mine?

   ii. a blank space?

   iii. the number k?

(b) The first square you picked revealed the number k. For your next move, you want to minimize the probability of picking a mine. Should you pick a square adjacent to your first pick, or a different square? Your answer should depend on the value of k.

(c) Your first move resulted in the number 1. You pick the square to the right for your next move. What is the probability that this square reveals the number 4?