1 Inequality Practice

(a) X is a random variable such that $X > -5$ and $\mathbb{E}[X] = -3$. Find an upper bound for the probability of X being greater than or equal to -1.

(b) You roll a die 100 times. Let Y be the sum of the numbers that appear on the die throughout the 100 rolls. Use Chebyshev's inequality to bound the probability of the sum Y being greater than 400 or less than 300.

2 Tightness of Inequalities

(a) Show by example that Markov's inequality is tight; that is, show that given $k > 0$, there exists a discrete non-negative random variable X such that $\mathbb{P}(X \geq k) = \mathbb{E}[X]/k$.

(b) Show by example that Chebyshev's inequality is tight; that is, show that given $k \geq 1$, there exists a random variable X such that $\mathbb{P}(|X - \mathbb{E}[X]| \geq k\sigma) = 1/k^2$, where $\sigma^2 = \operatorname{var} X$.

3 Working with the Law of Large Numbers

(a) A fair coin is tossed and you win a prize if there are more than 60% heads. Which is better: 10 tosses or 100 tosses? Explain.

(b) A fair coin is tossed and you win a prize if there are more than 40% heads. Which is better: 10 tosses or 100 tosses? Explain.

(c) A coin is tossed and you win a prize if there are between 40% and 60% heads. Which is better: 10 tosses or 100 tosses? Explain.

(d) A coin is tossed and you win a prize if there are exactly 50% heads. Which is better: 10 tosses or 100 tosses? Explain.

4 Vegas

On the planet Vegas, everyone carries a coin. Many people are honest and carry a fair coin (heads on one side and tails on the other), but a fraction p of them cheat and carry a trick coin with heads on both sides. You want to estimate p with the following experiment: you pick a random sample of n people and ask each one to flip his or her coin. Assume that each person is independently likely to carry a fair or a trick coin.

1. Given the results of your experiment, how should you estimate p?
2. How many people do you need to ask to be 95% sure that your answer is off by at most 0.05?