

1 Continuous Computations

Let X be a continuous random variable whose pdf is cx^3 (for some constant c) in the range $0 \leq x \leq 1$, and is 0 outside this range.

(a) Find c .

(b) Find $\mathbb{P}[1/3 \leq X \leq 2/3 \mid X \leq 1/2]$.

(c) Find $\mathbb{E}(X)$.

(d) Find $\text{var}(X)$.

2 Lunch Meeting

Alice and Bob agree to try to meet for lunch between 12 PM and 1 PM at their favorite sushi restaurant. Being extremely busy, they are unable to specify their arrival times exactly, and can say only that each of them will arrive (independently) at a time that is uniformly distributed within the hour. In order to avoid wasting precious time, if the other person is not there when they arrive they agree to wait exactly fifteen minutes before leaving. What is the probability that they will actually meet for lunch?

3 Exponential Distributions: Lightbulbs

A brand new lightbulb has just been installed in our classroom, and you know the life span of a lightbulb is exponentially distributed with a mean of 50 days.

- (a) Suppose an electrician is scheduled to check on the lightbulb in 30 days and replace it if it is broken. What is the probability that the electrician will find the bulb broken?

- (b) Suppose the electrician finds the bulb broken and replaces it with a new one. What is the probability that the new bulb will last at least 30 days?

- (c) Suppose the electrician finds the bulb in working condition and leaves. What is the probability that the bulb will last at least another 30 days?

4 First Exponential to Die

Let X and Y be $\text{Exponential}(\lambda_1)$ and $\text{Exponential}(\lambda_2)$ respectively, independent. What is

$$\mathbb{P}(\min(X, Y) = X),$$

the probability that the first of the two to die is X ?