1 Probabilistically Buying Probability Books

Chuck will go shopping for probability books for K hours. Here, K is a random variable and is equally likely to be 1, 2, or 3. The number of books N that he buys is random and depends on how long he shops. We are told that

$$\mathbb{P}[N = n | K = k] = \begin{cases} \frac{c}{k} & \text{for } n = 1, \ldots, k \\ 0 & \text{otherwise} \end{cases}$$

for some constant c.

(a) Compute c.

(b) Find the joint distribution of K and N.

(c) Find the marginal distribution of N.

2 Joint Distributions

(a) Give examples of joint distribution over discrete random variables X and Y such that $\mathbb{E}[XY] \neq \mathbb{E}[X]\mathbb{E}[Y]$.

(b) Give examples of joint distribution over discrete random variables X and Y such that $\mathbb{E}[XY] = 0$, $\mathbb{E}[X] = 0$, and $\mathbb{E}[Y] = 0$, but X and Y are not independent.

(c) Suppose that X_i, $i = 1, \ldots, n$ are binary-valued random variables. How many parameters are required to parameterize the joint distribution $\mathbb{P}(X_1 = x_1, \ldots, X_n = x_n)$?

(d) Continuing from the previous part, suppose that all X_is are independent. How many parameters are required to parameterize the joint distribution?

3 Binomial Conditioning

Let $n \in \mathbb{Z}_+$ and $p, q \in [0, 1]$. Let $X \sim \text{Binomial}(n, p)$ and suppose that conditioned on $X = x$, $Y \sim \text{Binomial}(x, q)$. What is the unconditional distribution of Y?