

1 Expectation and Variance

This problem will give you some practice calculating expectations and variances of random variables. Suppose that the random variable X takes on 3 values, 10, 25, 70. Suppose $\mathbb{P}[X = 10] = 0.5$, $\mathbb{P}[X = 25] = 0.2$, and $\mathbb{P}[X = 70] = 0.3$.

(a) What is $\mathbb{E}[X]$?

(b) What is $\mathbb{E}[X^2]$?

(c) What is $\text{var}(X)$?

2 Diversify Your Hand

You are dealt 13 cards from a standard 52 card deck. Let X be the number of distinct values in your hand. For instance, the hand (A, A, A, 2, 3, 4, 4, 5, 7, 9, 10, J, J) has 9 distinct values.

(a) Calculate $E[X]$.

(b) Calculate $\text{Var}[X]$.

3 Rolling Dice

- (a) If we roll a fair 6-sided die, what is the expected number of times we have to roll before we roll a 6? What is the variance?
- (b) Suppose we have two independent, fair n -sided dice labeled Die 1 and Die 2. If we roll the two dice until the value on Die 1 is smaller than the value on Die 2, what is the expected number of times that we roll? What is the variance?
- (c) Let $n = 6$, so we are back to fair 6-sided die. Suppose we roll Die 1 until a 6 comes up, and we roll Die 2 until a 6 comes up. Let X be a random variable representing the number of times Die 1 is rolled before getting a 6, and let Y be the corresponding random variable for Die 2. Compute $\mathbb{P}[\min(X, Y) = n]$ and $\mathbb{P}[X + Y = n]$, where n is an integer.

4 Fishy Computations

Use the Poisson distribution to answer these questions:

- (a) Suppose that on average, a fisherman catches 20 salmon per week. What is the probability that he will catch exactly 7 salmon this week?
- (b) Suppose that on average, you go to Fisherman's Wharf twice a year. What is the probability that you will go at most once in 2018?
- (c) Suppose that in March, on average, there are 5.7 boats that sail in Laguna Beach per day. What is the probability there will be *at least* 3 boats sailing throughout the *next two days* in Laguna?