

1 How Many Queens?

You shuffle a standard 52-card deck, before drawing the first three cards from the top of the pile. Let X denote the number of queens you draw.

- (a) What is $\mathbb{P}(X = 0)$?
- (b) What is $\mathbb{P}(X = 1)$?
- (c) What is $\mathbb{P}(X = 2)$?
- (d) What is $\mathbb{P}(X = 3)$?
- (e) Do the answers you computed in parts (a) through (d) add up to 1, as expected?
- (f) Compute $\mathbb{E}(X)$ from the definition of expectation.
- (g) Suppose we define indicators X_i , $1 \leq i \leq 3$, where X_i is the indicator variable that equals 1 if the i th card is a queen and 0 otherwise. Compute $\mathbb{E}(X)$ using linearity of expectation.
- (h) Are the X_i indicators independent? Does this affect your solution to part (g)?

2 More Aces in a Deck

There are four aces in a deck. Suppose you shuffle the deck; define the random variables:

X_1 = number of non-ace cards before the first ace

X_2 = number of non-ace cards between the first and second ace

X_3 = number of non-ace cards between the second and third ace

X_4 = number of non-ace cards between the third and fourth ace

X_5 = number of non-ace cards after the fourth ace

1. What is $X_1 + X_2 + X_3 + X_4 + X_5$?
2. Argue that the X_i random variables all have the same distribution. Are they independent?

3 Head Count

Consider a coin with $\mathbb{P}(\text{Heads}) = 2/5$. Suppose you flip the coin 20 times, and define X to be the number of heads.

(a) What is the distribution of X ?

(b) What is $\mathbb{P}(X = 7)$?

(c) What is $\mathbb{P}(X \geq 1)$?

(d) What is $\mathbb{P}(12 \leq X \leq 14)$?