# 1   Count It!

For each of the following collections, determine and briefly explain whether it is finite, countably infinite (like the natural numbers), or uncountably infinite (like the reals):

(a) $\mathbb{N}$, the set of all natural numbers.

(b) $\mathbb{Z}$, the set of all integers.

(c) $\mathbb{Q}$, the set of all rational numbers.

(d) $\mathbb{R}$, the set of all real numbers.

(e) The integers which divide 8.

(f) The integers which 8 divides.

(g) The functions from $\mathbb{N}$ to $\mathbb{N}$.

(h) Computer programs that halt.

(i) Numbers that are the roots of nonzero polynomials with integer coefficients.

# 2   Computability

Decide whether the following statements are true or false. Please justify your answers.

(a) The problem of determining whether a program halts in time $2^{n^2}$ on an input of size $n$ is undecidable.

(b) There is no computer program `Line` which takes a program $P$, an input $x$, and a line number $L$, and determines whether the $L^{\text{th}}$ line of code is executed when the program $P$ is run on the input $x$.

# 3  Hello World!

Determine the computability of the following tasks. If it's not computable, write a reduction or self-reference proof. If it is, write the program.

(a) You want to determine whether a program $P$ on input $x$ prints "Hello World!". Is there a computer program that can perform this task? Justify your answer.

(b) You want to determine whether a program $P$ prints "Hello World!" before running the $k$th line in the program. Is there a computer program that can perform this task? Justify your answer.

(c) You want to determine whether a program $P$ prints "Hello World!" in the first $k$ steps of its execution. Is there a computer program that can perform this task? Justify your answer.