Today.

Finish up undecidability.

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Counting.

Programs and Diagonalization.

Write me a program checker!

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Check that the compiler works!

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How about.. Check that the compiler terminates on a certain input.

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Run P on I and check!

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How long do you wait?

Something about infinity here, maybe?

Halt does not exist.

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Theorem: There is no program HALT.

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Proof:

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Proof: Assume there is a program $HALT(\cdot, \cdot)$.

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Contradiction. Program HALT does not exist!

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Another view of proof: diagonalization.

Any program is a fixed length string.

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Fixed length strings are enumerable.

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Program halts or not any input, which is a string.

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	P_1	P_2	P_3	$\cdots$
P_1	H	H	L	$\cdots$
P_2	L	L	H	$\cdots$
P_3	L	H	H	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

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Halt - diagonal.

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and is different from every P_i on the diagonal.

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Any program is a fixed length string.
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$$
\begin{array}{c|cccc}
 & P_1 & P_2 & P_3 & \cdots \\
\hline
 & & & & \\
P_1 & \color{blue}{H} & H & L & \cdots \\
P_2 & L & \color{blue}{L} & H & \cdots \\
P_3 & L & H & \color{blue}{H} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{array}
$$

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Discussion of Proof.

Undecidability:

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Undecidability:
HALT(P) - does not exist.

Discussion of Proof.

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Why not?

Discussion of Proof.

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Argue directly by saying Turing(Turing) neither halts nor runs forever.

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Argue directly by saying Turing(Turing) neither halts nor runs forever.

Really Same: Says there is no text which can be Turing.

Notes.

Fairly quick. Right?

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CS 61C + one slide

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CS 61C + one slide $\implies$ undecidability of halting.

Undecidable questions about Programs.

Does a program, P, print "Hello World"?

Undecidable questions about Programs.

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How?

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Does a program, P, print "Hello World"?
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Solve HALT(P,I):
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Because programs are text.

Other example undecidable problems.

Can a set of notched tiles tile the infinite plane?

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Undecidability for Diophantine set of equations
$\implies$ no program can take any set of integer equations and
always corectly output whether it has an integer solution.

Comments: undecidability.

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Computation as a lens

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Today: Quantum computing, evolution models, models of the brain, complexity of Nash equilibria, ...

Your future (in this course).

What's to come?

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What's to come? Probability.

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A bag contains:

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A bag contains:

What is the chance that a ball taken from the bag is blue?

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Count blue.

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Count blue. Count total.

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Babak $\equiv$ "Bob" Back.

Outline: basics

1. Counting.
2. Tree
3. Rules of Counting
4. Sample with/without replacement where order does/doesn't matter.

Probability is soon..but first let’s count.

Count?

How many outcomes possible for k coin tosses?
How many poker hands?
How many handshakes for n people?
How many diagonals in a convex polygon?
How many 10 digit numbers?
How many 10 digit numbers without repetition?

Using a tree..

How many 3-bit strings?

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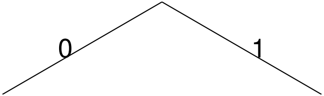

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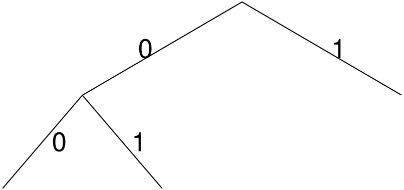

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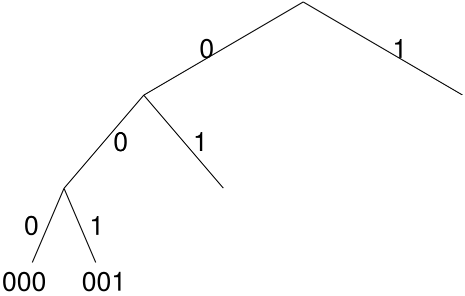

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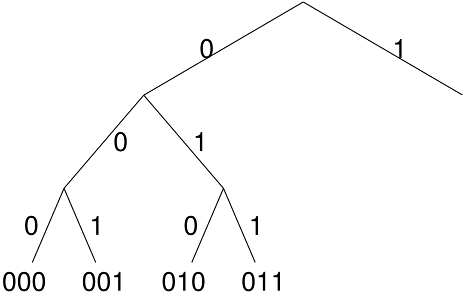

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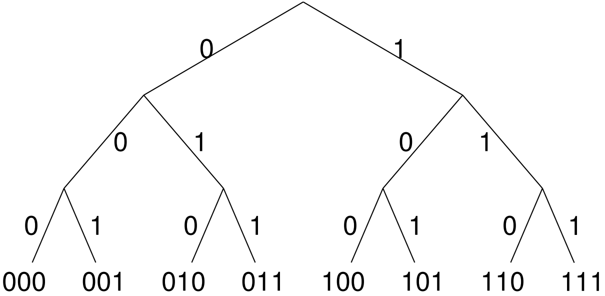

8 leaves which is $2 \times 2 \times 2$.

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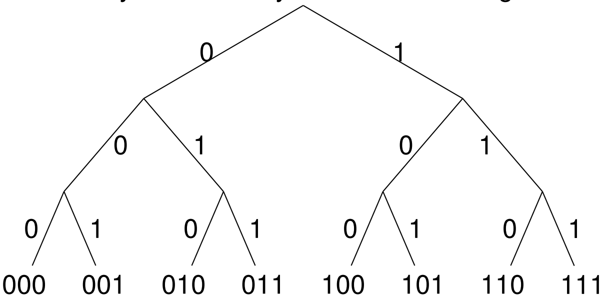

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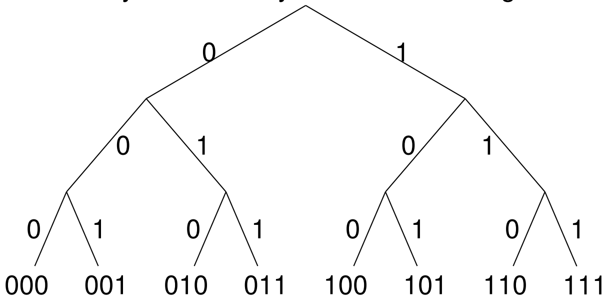

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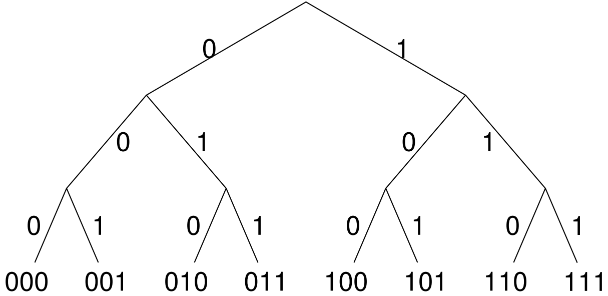

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First Rule of Counting: Product Rule

Objects made by choosing from n_1, then n_2, ..., then n_k
the number of objects is $n_1 \times n_2 \cdots \times n_k$.

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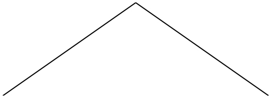

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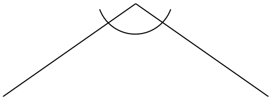

n_1

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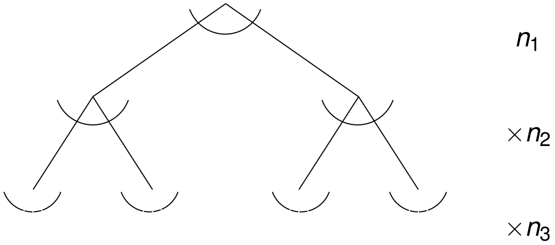

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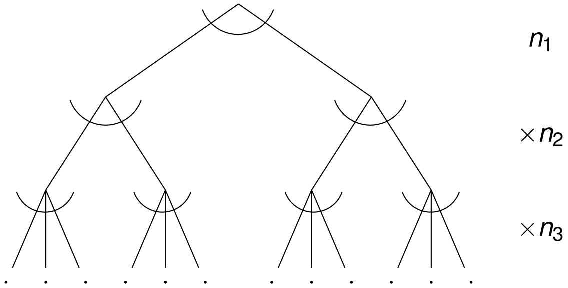

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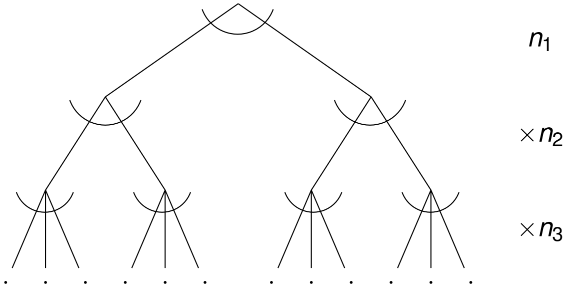

In picture, $2 \times 2 \times 3 = 12$!

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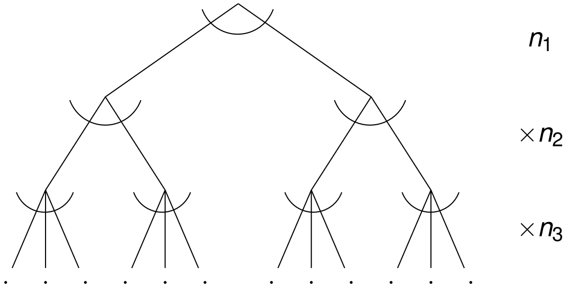

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How many outcomes possible for k coin tosses?

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2 ways for first choice,

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...

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How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
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2×2

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$10 \times$

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

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If no. Then $(m-1)m^{n-1}$.

Functions, polynomials.

How many functions f mapping S to T?

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Questions?

Permutations.

[1] By definition: $0! = 1$.

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How many 10 digit numbers **without repeating a digit**?

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How many 10 digit numbers **without repeating a digit**?

10 ways for first, 9 ways for second, 8 ways for third,

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So total number is $|S| \times |S|-1 \cdots 1 = |S|!$
A one-to-one function is a permutation!

Counting sets..when order doesn't matter.

How many poker hands?

[2]When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$

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Counting sets..when order doesn't matter.

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Are $A, K, Q, 10, J$ of spades
and $10, J, Q, K, A$ of spades the same?

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Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.[2]

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Number of orderings for a poker hand: "5!"

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Number of orderings for a poker hand: "5!"
(The "!" means factorial, not Exclamation.)

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$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$

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Number of orderings for a poker hand: "5!"

Can write as...
$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$
$$\frac{52!}{5! \times 47!}$$

[2]When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$???

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Number of orderings for a poker hand: "5!"

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$$\frac{52!}{5! \times 47!}$$

Generic: ways to choose 5 out of 52 possibilities.

---

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Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.

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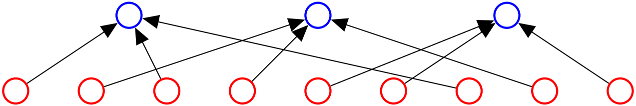

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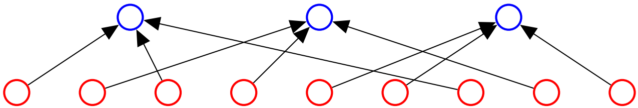

How many red nodes (ordered objects)?

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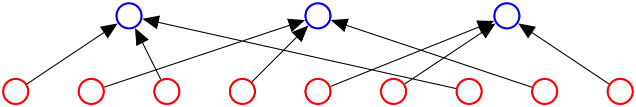

How many red nodes (ordered objects)? 9.

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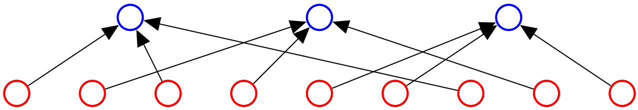

How many red nodes (ordered objects)? 9.

How many red nodes mapped to one blue node?

Ordered to unordered.

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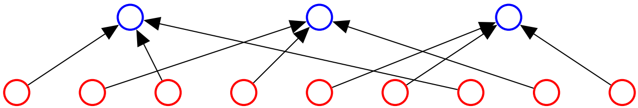

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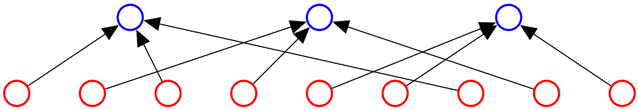

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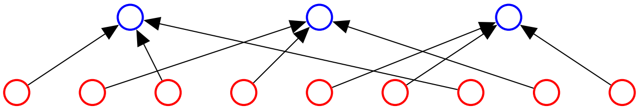

How many red nodes (ordered objects)? 9.

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How many blue nodes (unordered objects)? $\frac{9}{3}$

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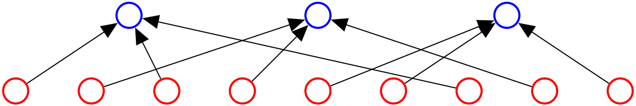

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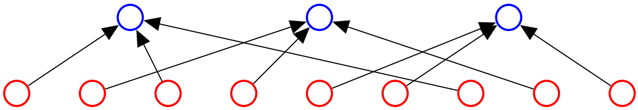

How many red nodes (ordered objects)? 9.

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How many poker deals?

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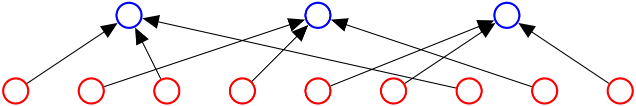

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How many poker deals? $52 \cdot 51 \cdot 50 \cdot 49 \cdot 48$.

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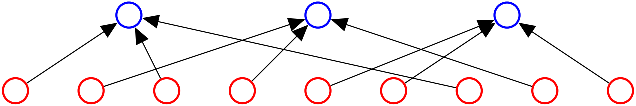

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How many poker deals per hand?

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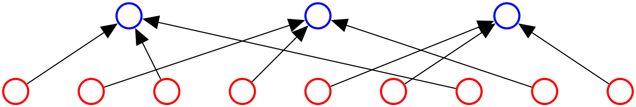

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How many poker deals per hand?
  Map each deal to ordered deal:

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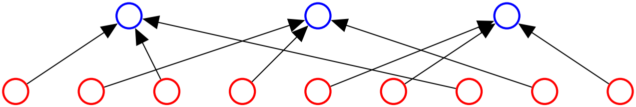

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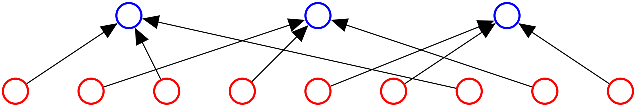

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How many blue nodes (unordered objects)? $\frac{9}{3} = 3$.

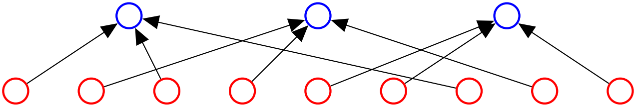
How many poker deals? $52 \cdot 51 \cdot 50 \cdot 49 \cdot 48$.

How many poker deals per hand?
Map each deal to ordered deal: $5!$

How many poker hands?

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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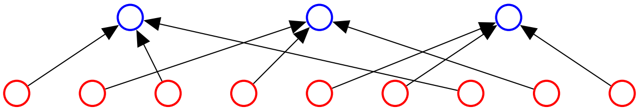

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Questions?

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Choose 2 out of n?

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Choose k **out of** n?

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Notation: $\binom{n}{k}$ **and pronounced "**n **choose** k**."**

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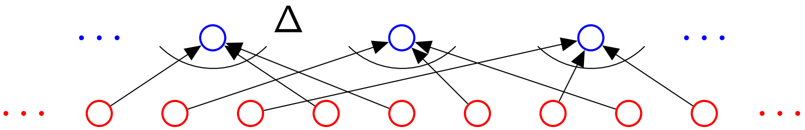
Second rule: when order doesn't matter divide...



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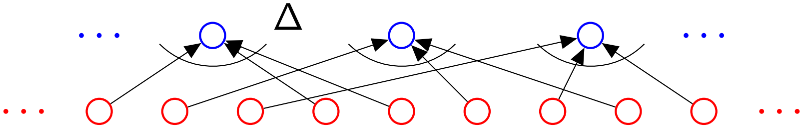

3 card Poker deals: 52

Example: Visualize the proof..

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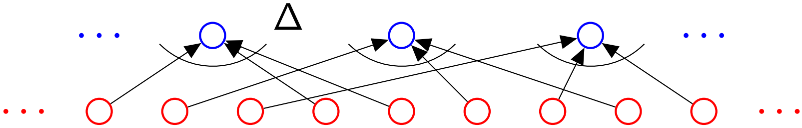

3 card Poker deals: 52×51

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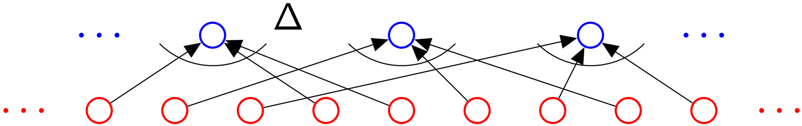

3 card Poker deals: $52 \times 51 \times 50$

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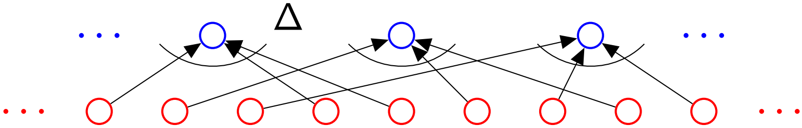

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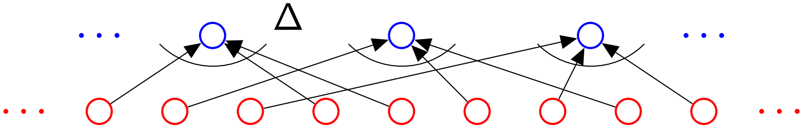

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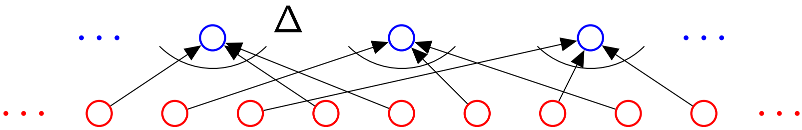

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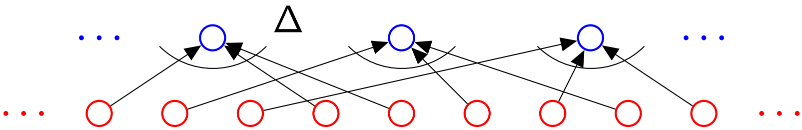

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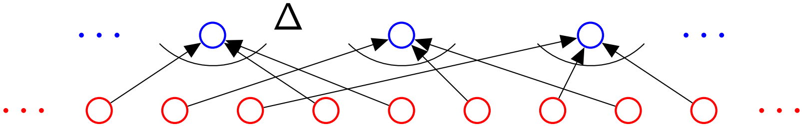

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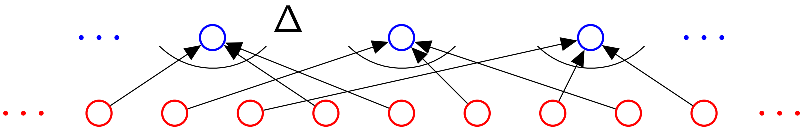

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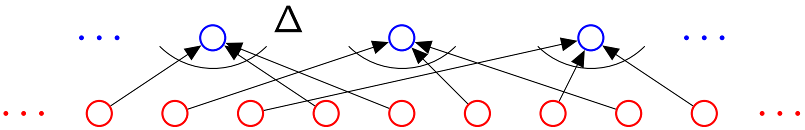

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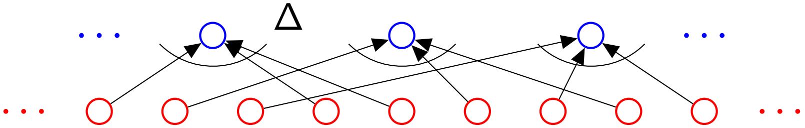

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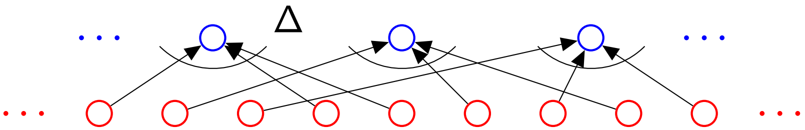

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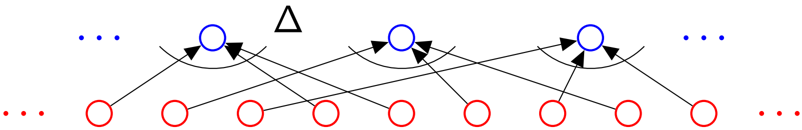

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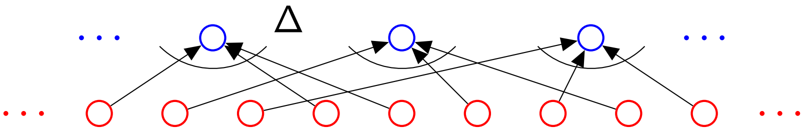

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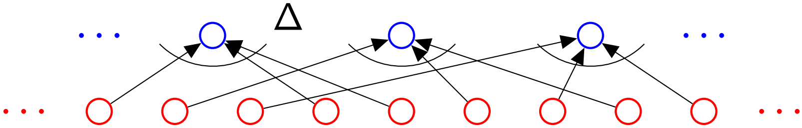

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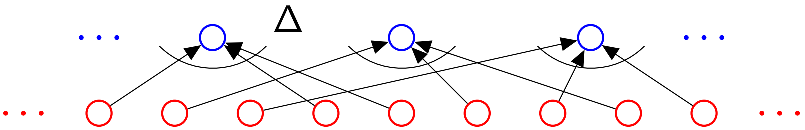

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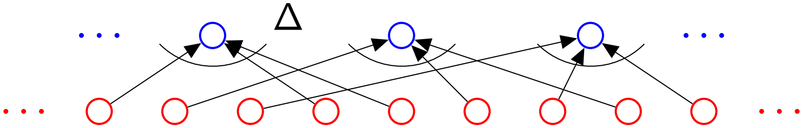

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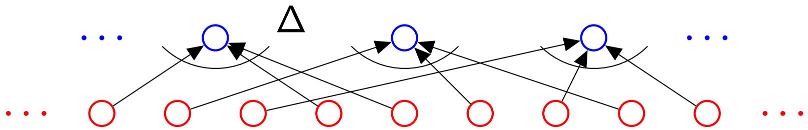

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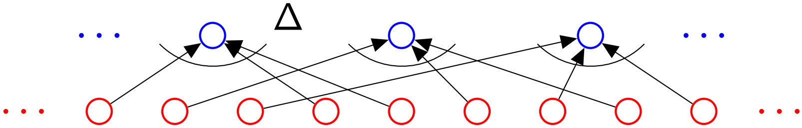

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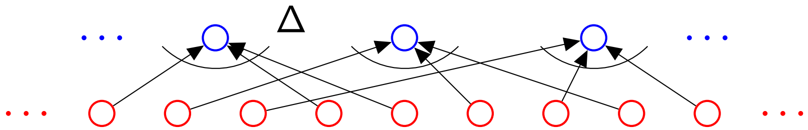

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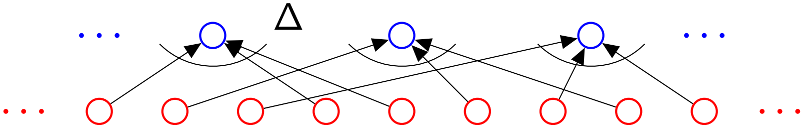

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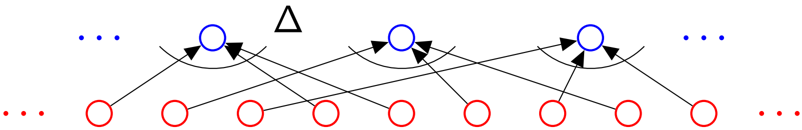

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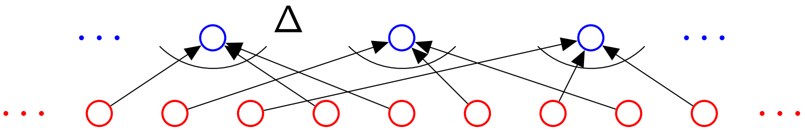

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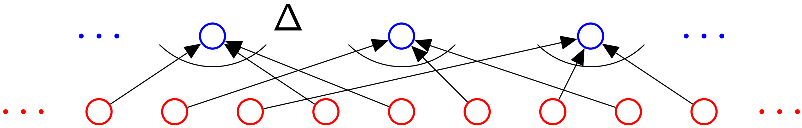

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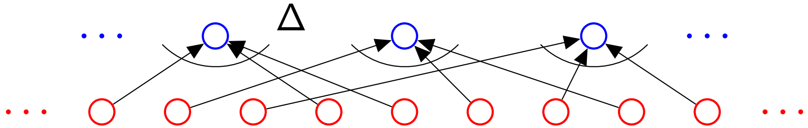

Orderings of ANAGRAM?

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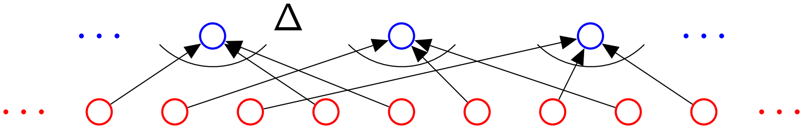

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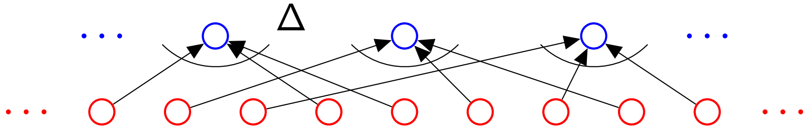

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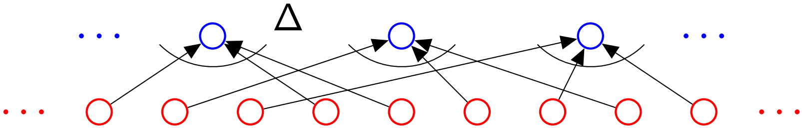

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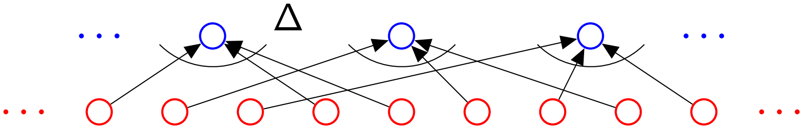

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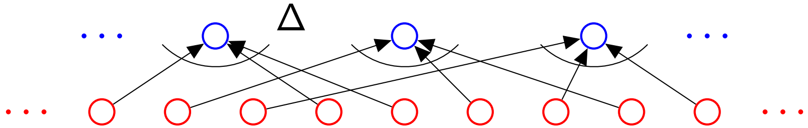

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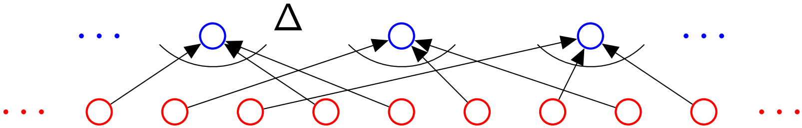

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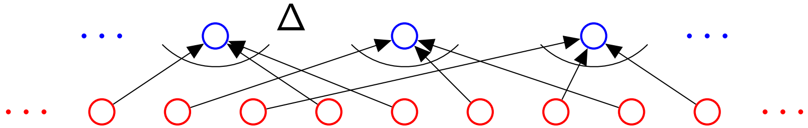

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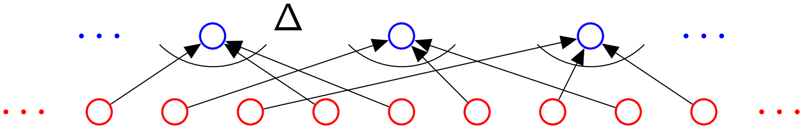

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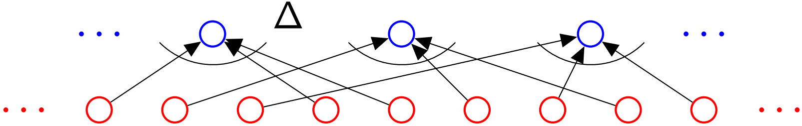

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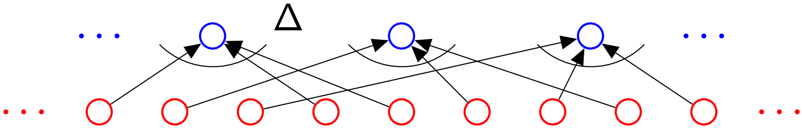

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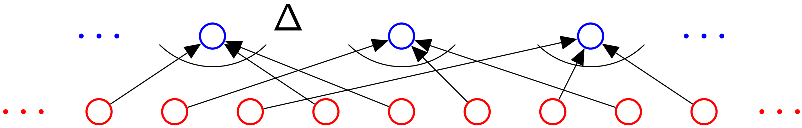

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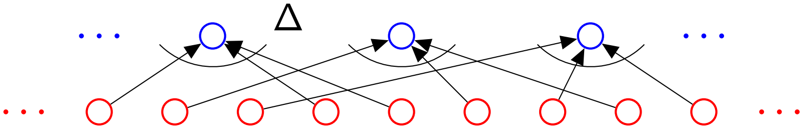

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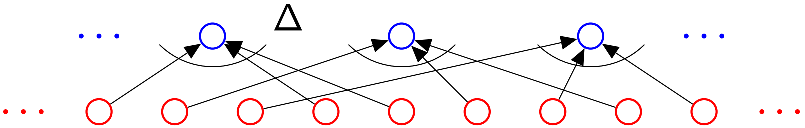

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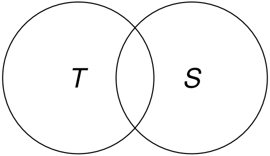

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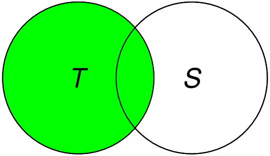

In T. $\implies$ $|T|$

Simple Inclusion/Exclusion

Sum Rule: For disjoint sets S **and** T**,** $|S \cup T| = |S| + |T|$

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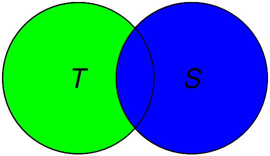

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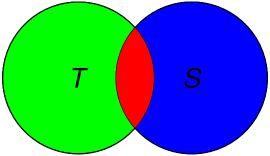

$S \cap T$

In T. $\implies$ $|T|$
In S. $\implies$ $+$ $|S|$
Elements in $S \cap T$ are counted twice.

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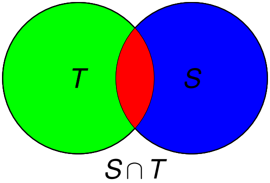

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Subtract. $\implies -|S \cap T|$

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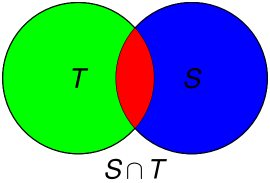

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Using Inclusion/Exclusion.

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T = phone numbers with 7 as second digit. $|T| = 10^9$.

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Answer: $|S| + |T| - |S \cap T| = 10^9 + 10^9 - 10^8$.

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Make object out of sequence of choices.

n_i - number of choices for choice i.

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Sum Rule:
  Size of union of disjoint sets is sum of sizes.

Inclusion/Exclusion:
  Size of union of sets is sum of sizes
    minus the size of intersection.