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Arithmetic $\pmod p \implies$ work with $O(\log p)$ bit numbers.

Erasure Codes.

Satellite

GPS device

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3 packet message.

GPS device

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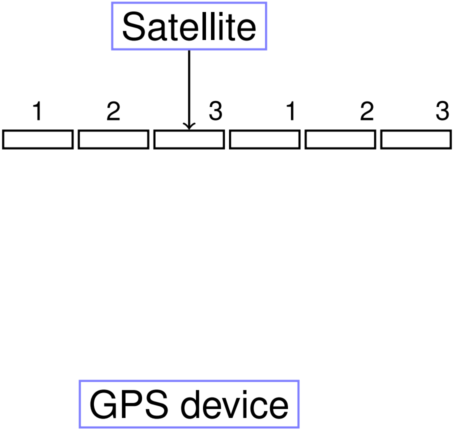
Satellite

3 packet message.

Lose 3 out 6 packets.

GPS device

Erasure Codes.



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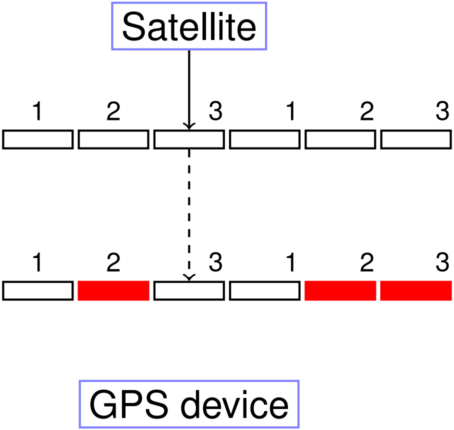

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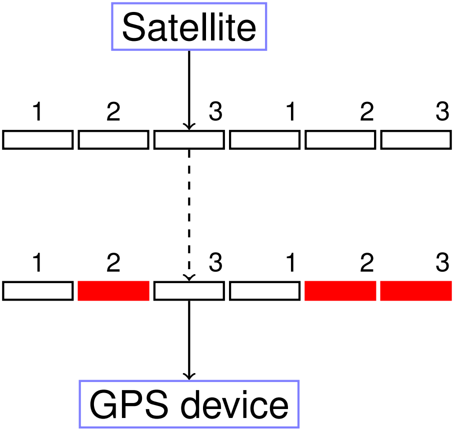

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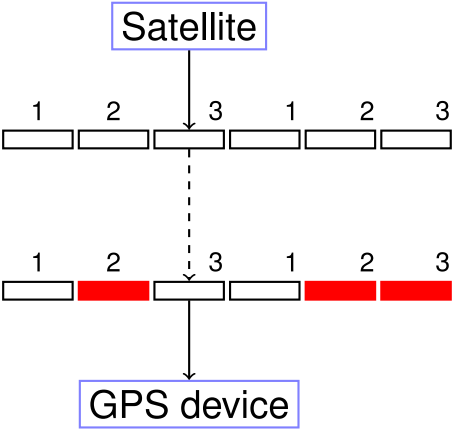

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Gets packets 1,1,and 3.

Solution Idea.

n packet message, channel that loses k packets.

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Representing vector (message) in different basis.

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Many bases!

The Scheme

Problem: Want to send a message with n packets.

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GPS device

Erasure Codes.

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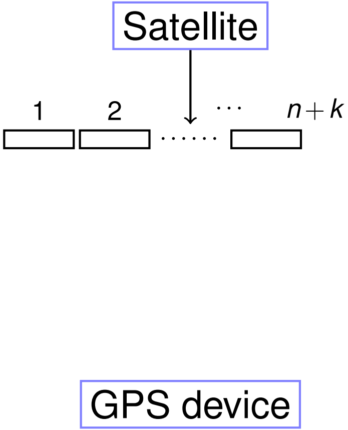
Erasure Codes.

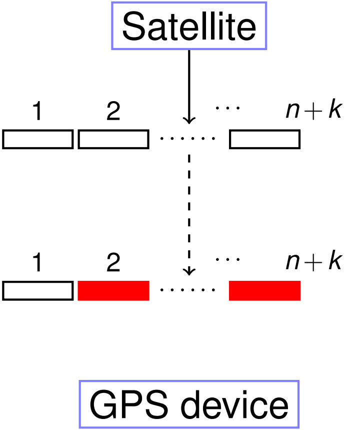


n packet message.

So send $n+k$ points on polynomial.

Lose k packets.

Erasure Codes.



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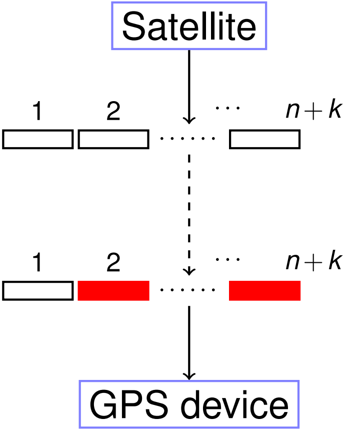

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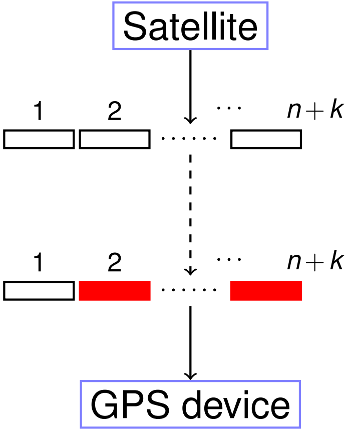

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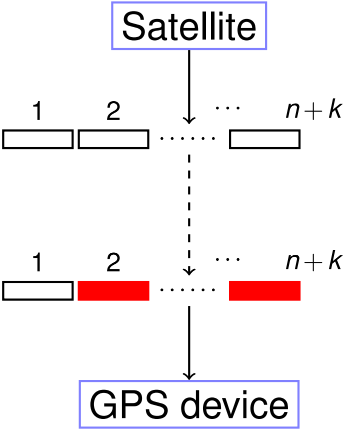

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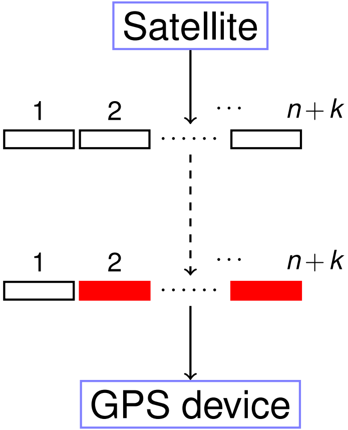

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Optimal.

Polynomials.

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- ..give Secret Sharing.

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Error Correction:

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Error Correction:

Noisy Channel: corrupts k packets. (rather than loses.)

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Additional Challenge: Finding which packets are corrupt.

Error Correction

Satellite

GPS device

Which one was corrupted?

Error Correction

Satellite

3 packet message.

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Satellite

3 packet message.

Corrupts 1 packets.

GPS device

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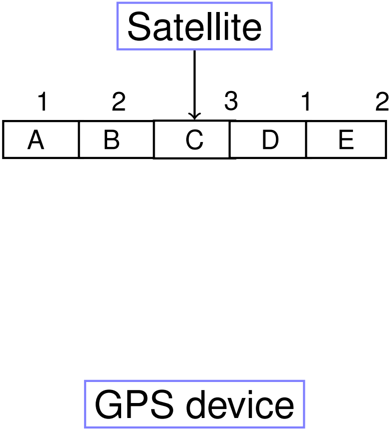
Error Correction



Which one was corrupted?

3 packet message. Send 5.

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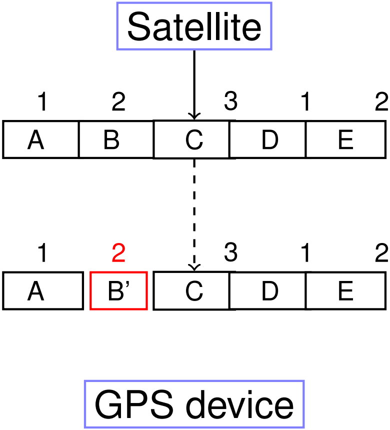
Error Correction



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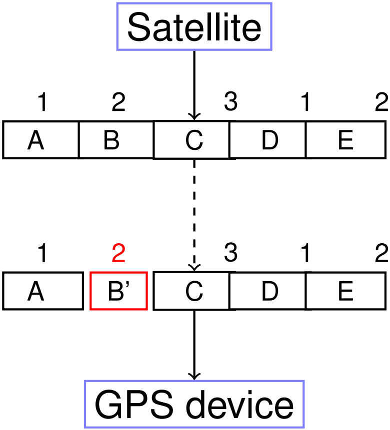

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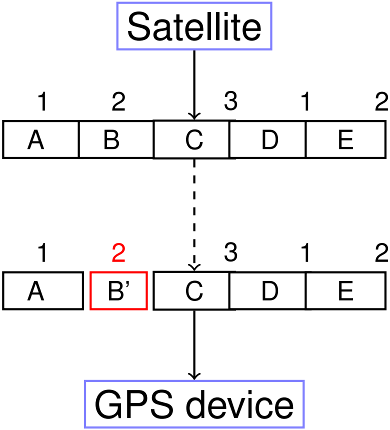

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Problem: Communicate n packets $m_1, \ldots, m_n$
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Properties: proof.

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$P(x)$: degree $n-1$ polynomial.
Send $P(1),\ldots,P(n+2k)$
Receive $R(1),\ldots,R(n+2k)$
At most k i's where $P(i) \neq R(i)$.

Properties:

(1) $P(i) = R(i)$ for at least $n+k$ points i,
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Degree 3 $\implies P(x) = Q(x)$

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$P(i) = R(i)$ for $n + k = 3 + 1 = 4$ points.

Slow solution.

Brute Force:
For each subset of $n+k$ points

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Reconstructs $P(x)$ and only $P(x)$!!

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Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

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All equations..

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Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

All equations..

$$
\begin{aligned}
p_2 + p_1 + p_0 &\equiv 3 \pmod 7 \\
4p_2 + 2p_1 + p_0 &\equiv 1 \pmod 7 \\
2p_2 + 3p_1 + p_0 &\equiv 6 \pmod 7 \\
2p_2 + 4p_1 + p_0 &\equiv 0 \pmod 7 \\
4p_2 + 5p_1 + p_0 &\equiv 3 \pmod 7
\end{aligned}
$$

Assume point 1 is wrong and solve..no consistent solution!

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Assume point 1 is wrong and solve..no consistent solution!
Assume point 2 is wrong

Example.

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$$

Assume point 1 is wrong and solve..no consistent solution!
Assume point 2 is wrong and solve...consistent solution!

In general..

$P(x) = p_{n-1}x^{n-1} + \cdots p_0$ and receive $R(1), \ldots R(m = n + 2k)$.

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Error!!

In general..

$P(x) = p_{n-1}x^{n-1} + \cdots p_0$ and receive $R(1), \ldots R(m = n + 2k)$.

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p_{n-1} + \cdots p_0 &\equiv R(1) \pmod p \\
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&. \\
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&. \\
p_{n-1}(m)^{n-1} + \cdots p_0 &\equiv R(m) \pmod p
\end{aligned}$$

Error!! .... Where???

In general..

$P(x) = p_{n-1}x^{n-1} + \cdots p_0$ and receive $R(1), \ldots R(m = n + 2k)$.

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\end{aligned}
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Error!! .... Where???
Could be anywhere!!!

In general..

$P(x) = p_{n-1}x^{n-1} + \cdots p_0$ and receive $R(1), \ldots R(m = n + 2k)$.

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Error!! .... Where???
Could be anywhere!!! ...so try everywhere.

In general..

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\end{aligned}
$$

Error!! .... Where???
Could be anywhere!!! ...so try everywhere.
Runtime: $\binom{n+2k}{k}$ possibilitities.

In general..

$P(x) = p_{n-1}x^{n-1} + \cdots p_0$ and receive $R(1), \ldots R(m = n + 2k)$.

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p_{n-1} + \cdots p_0 &\equiv R(1) \pmod{p} \\
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Error!! .... Where???
Could be anywhere!!! ...so try everywhere.
Runtime: $\binom{n+2k}{k}$ possibilitities.

Something like $(n/k)^k$ ...Exponential in k!

In general..

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$$\begin{aligned} p_{n-1} + \cdots p_0 &\equiv R(1) \pmod p \\ p_{n-1}2^{n-1} + \cdots p_0 &\equiv R(2) \pmod p \\ &\cdot \\ p_{n-1}i^{n-1} + \cdots p_0 &\equiv R(i) \pmod p \\ &\cdot \\ p_{n-1}(m)^{n-1} + \cdots p_0 &\equiv R(m) \pmod p \end{aligned}$$

Error!! .... Where???
Could be anywhere!!! ...so try everywhere.
Runtime: $\binom{n+2k}{k}$ possibilitities.

Something like $(n/k)^k$ ...Exponential in k!

How do we find where the bad packets are efficiently?!?!?!

Ditty...

Oh where, Oh where

Ditty...

Oh where, Oh where
has my little dog gone?

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Ditty...

Oh where, Oh where
has my little dog gone?
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With his ears cut short
And his tail cut long
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Oh where, Oh where

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone..

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

With the polynomial well put

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

With the polynomial well put
But the channel a bit wrong

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

With the polynomial well put
But the channel a bit wrong
Where, oh where do we look?

Where oh where can my bad packets be?

$$(p_{n-1} + \cdots p_0) \quad \equiv \quad R(1) \qquad \pmod{p}$$

Where oh where can my bad packets be?

$$
\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
(p_{n-1} 2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\
&\vdots \\
(p_{n-1} (m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}
\end{aligned}
$$

Where oh where can my bad packets be?

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\begin{aligned}
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\end{aligned}
$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

Where oh where can my bad packets be?

$$
\begin{array}{rrcll}
 & (p_{n-1} + \cdots p_0) & \equiv & R(1) & \pmod{p} \\
\mathbf{0}\times & (p_{n-1}2^{n-1} + \cdots p_0) & \equiv & R(2) & \pmod{p} \\
 & & \vdots & & \\
 & (p_{n-1}(m)^{n-1} + \cdots p_0) & \equiv & R(n+2k) & \pmod{p}
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Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
All equations satisfied!!!!!

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Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
All equations satisfied!!!!!

But which equations should we multiply by 0?

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Where oh where can my bad packets be?

$$\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) && \pmod{p} \\
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We will use a polynomial!!!

Where oh where can my bad packets be?

$$\begin{array}{rcll} (p_{n-1} + \cdots p_0) & \equiv & R(1) & \pmod{p} \\ (p_{n-1}2^{n-1} + \cdots p_0) & \equiv & R(2) & \pmod{p} \\ & \vdots & & \\ (p_{n-1}(m)^{n-1} + \cdots p_0) & \equiv & R(n+2k) & \pmod{p} \end{array}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
All equations satisfied!!!!!

But which equations should we multiply by 0? Where oh where...??

We will use a polynomial!!! That we don't know.

Where oh where can my bad packets be?

$$
\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
(p_{n-1} 2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\
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Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
All equations satisfied!!!!!

But which equations should we multiply by 0? Where oh where...??

We will use a polynomial!!! That we don't know. But can find!

Where oh where can my bad packets be?

$$\begin{aligned}
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Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

Where oh where can my bad packets be?

$$\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
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Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)$

Where oh where can my bad packets be?

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\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
(p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\
&\vdots \\
(p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}
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Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
All equations satisfied!!!!!

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Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)(x - e_2)$

Where oh where can my bad packets be?

$$\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
(p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\
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\end{aligned}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
All equations satisfied!!!!!

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Error locator polynomial: $E(x) = (x - e_1)(x - e_2)\ldots$

Where oh where can my bad packets be?

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Error locator polynomial: $E(x) = (x - e_1)(x - e_2) \ldots (x - e_k)$.

Where oh where can my bad packets be?

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\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
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Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)(x - e_2)\ldots(x - e_k)$.

$E(i) = 0$ if and only if $e_j = i$ for some j

Where oh where can my bad packets be?

$$
\begin{aligned}
E(1)(p_{n-1} + \cdots p_0) &\equiv R(1)E(1) \pmod{p} \\
E(2)(p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2)E(2) \pmod{p} \\
&\vdots \\
E(m)(p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k)E(m) \pmod{p}
\end{aligned}
$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
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$E(i) = 0$ if and only if $e_j = i$ for some j

Multiply equations by $E(\cdot)$.

Where oh where can my bad packets be?

$$\begin{aligned}
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\end{aligned}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
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Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)(x - e_2)\ldots(x - e_k)$.

$E(i) = 0$ if and only if $e_j = i$ for some j

Multiply equations by $E(\cdot)$. (Above E(x) = (x-2).)

Where oh where can my bad packets be?

$$\begin{aligned} E(1)(p_{n-1} + \cdots p_0) &\equiv R(1)E(1) \pmod{p} \\ E(2)(p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2)E(2) \pmod{p} \\ &\vdots \\ E(m)(p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k)E(m) \pmod{p} \end{aligned}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
All equations satisfied!!!!!

But which equations should we multiply by 0? Where oh where...??

We will use a polynomial!!! That we don't know. But can find!

Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)(x - e_2)\ldots(x - e_k)$.

$E(i) = 0$ if and only if $e_j = i$ for some j

Multiply equations by $E(\cdot)$. (Above E(x) = (x-2).)

All equations satisfied!!

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2 x^2 + p_1 x + p_0$ that contains $n + k = 3 + 1$ points.

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2 x^2 + p_1 x + p_0$ that contains $n + k = 3 + 1$ points.

Plugin points...

$$
\begin{aligned}
(p_2 + p_1 + p_0) &\equiv (3) && \pmod 7 \\
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Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2 x^2 + p_1 x + p_0$ that contains $n + k = 3 + 1$ points.

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4 unknowns (p_0, p_1, p_2 and e), 5 nonlinear equations.

..turn their heads each day,

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\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod p \\
&\vdots \\
(p_{n-1} i^{n-1} + \cdots p_0) &\equiv R(i) \pmod p \\
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(p_{n-1} (n+2k)^{n-1} + \cdots p_0) &\equiv R(m) \pmod p
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E(1)(p_{n-1} + \cdots p_0) &\equiv R(1)E(1) \pmod{p} \\
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and linear in a_i and coefficients of $E(x)$!

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Solving for $Q(x)$ and $E(x)$...

For all points $1, \ldots, i, n+2k = m$,

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Solve for coefficients of $Q(x)$ and $E(x)$.

Solving for $Q(x)$ and $E(x)$...and $P(x)$

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$$\text{Find } P(x) = Q(x)/E(x).$$

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Solve for coefficients of $Q(x)$ and $E(x)$.

Find $P(x) = Q(x)/E(x)$.

Solving for $Q(x)$ and $E(x)$...and $P(x)$

For all points $1,\ldots,i,n+2k=m$,

$$Q(i) = R(i)E(i) \pmod{p}$$

Gives $n+2k$ linear equations.

$$\begin{aligned}
a_{n+k-1} + \ldots a_0 &\equiv R(1)(1 + b_{k-1} \cdots b_0) \pmod{p} \\
a_{n+k-1}(2)^{n+k-1} + \ldots a_0 &\equiv R(2)((2)^k + b_{k-1}(2)^{k-1} \cdots b_0) \pmod{p} \\
&\vdots \\
a_{n+k-1}(m)^{n+k-1} + \ldots a_0 &\equiv R(m)((m)^k + b_{k-1}(m)^{k-1} \cdots b_0) \pmod{p}
\end{aligned}$$

..and $n+2k$ unknown coefficients of $Q(x)$ and $E(x)$!

Solve for coefficients of $Q(x)$ and $E(x)$.

Find $P(x) = Q(x)/E(x)$.

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

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$$a_3 + a_2 + a_1 + a_0 \equiv 3(1 - b_0) \pmod{7}$$

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$$\begin{aligned} a_3 + a_2 + a_1 + a_0 &\equiv 3(1 - b_0) \pmod 7 \\ a_3 + 4a_2 + 2a_1 + a_0 &\equiv 1(2 - b_0) \pmod 7 \end{aligned}$$

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6a_3 + 2a_2 + 3a_1 + a_0 &\equiv 6(3 - b_0) \pmod 7 \\
a_3 + 2a_2 + 4a_1 + a_0 &\equiv 0(4 - b_0) \pmod 7 \\
6a_3 + 4a_2 + 5a_1 + a_0 &\equiv 3(5 - b_0) \pmod 7
\end{aligned}
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$a_3 = 1$, $a_2 = 6$, $a_1 = 6$, $a_0 = 5$ and $b_0 = 2$.

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$Q(x) = x^3 + 6x^2 + 6x + 5.$

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$E(x) = x - 2$.

Example: finishing up.

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$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
            _________________
x - 2 ) x^3  + 6 x^2 + 6 x + 5
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                 1 x^2
        ------------------
x - 2 ) x^3   + 6 x^2 + 6 x + 5
        x^3   - 2 x^2
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                  1 x^2
          -----------------
x - 2 )  x^3   + 6 x^2 + 6 x + 5
         x^3   - 2 x^2
         ----------
                  1 x^2 + 6 x + 5
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                1 x^2 + 1 x
          -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                1 x^2 + 6 x + 5
                1 x^2 - 2 x
```

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$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                 1 x^2 + 1 x
          -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                  1 x^2 + 1 x + 1
          -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                1 x^2 + 6 x + 5
                1 x^2 - 2 x
                ---------------
                          x + 5
                          x - 2
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                 1 x^2 + 1 x + 1
          -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
                           x - 2
                           -----
                               0
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                 1 x^2 + 1 x + 1
          -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
                           x - 2
                           -----
                               0
```

$P(x) = x^2 + x + 1$

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                  1 x^2 + 1 x + 1
          -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
        x^3  - 2 x^2
        ----------
                  1 x^2 + 6 x + 5
                  1 x^2 - 2 x
                  ---------------
                            x + 5
                            x - 2
                            -----
                                0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                  1 x^2 + 1 x + 1
          -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                  1 x^2 + 6 x + 5
                  1 x^2 - 2 x
                  ---------------
                            x + 5
                            x - 2
                            -----
                                0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$?

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                 1 x^2 + 1 x + 1
           -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
                           x - 2
                           -----
                               0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$? 1

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$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                 1 x^2 + 1 x + 1
          -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                 1 x^2 + 6 x + 5
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                 ---------------
                           x + 5
                           x - 2
                           -----
                               0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$? 1
Except at $x = 2$?

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                    1 x^2 + 1 x + 1
              -----------------
    x - 2 ) x^3  + 6 x^2 + 6 x + 5
            x^3  - 2 x^2
            ----------
                    1 x^2 + 6 x + 5
                    1 x^2 - 2 x
                    ---------------
                              x + 5
                              x - 2
                              -----
                                  0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$? 1
Except at $x = 2$? Hole there?

Error Correction: Berlekamp-Welsh

Message: $m_1, \ldots, m_n$.
Sender:

1. Form degree $n-1$ polynomial $P(x)$ where $P(i) = m_i$.
2. Send $P(1), \ldots, P(n+2k)$.

Receiver:

1. Receive $R(1), \ldots, R(n+2k)$.
2. Solve $n+2k$ equations, $Q(i) = E(i)R(i)$ to find $Q(x) = E(x)P(x)$ and $E(x)$.
3. Compute $P(x) = Q(x)/E(x)$.
4. Compute $P(1), \ldots, P(n)$.

Check your undersanding.

You have error locator polynomial!

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency? Sure. Only $n+2k$ values.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency? Sure. Only $n+2k$ values.
See where it is 0.

Hmmm...

Is there one and only one $P(x)$ from Berlekamp-Welsh procedure?

Hmmm...

Is there one and only one $P(x)$ from Berlekamp-Welsh procedure?

Existence: there is a $P(x)$ and $E(x)$ that satisfy equations.

Unique solution for $P(x)$

Uniqueness: any solution $Q'(x)$ and $E'(x)$ have

$$\frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)} = P(x). \tag{1}$$

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Uniqueness: any solution $Q'(x)$ and $E'(x)$ have

$$\frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)} = P(x). \qquad (1)$$

Proof:

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Proof:
We claim

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Proof:
We claim

$$Q'(x)E(x) = Q(x)E'(x) \text{ on } n+2k \text{ values of } x. \tag{2}$$

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Equation 2 implies 1:

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Equation 2 implies 1:

$Q'(x)E(x)$ and $Q(x)E'(x)$ are degree $n+2k-1$

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Equation 2 implies 1:

$Q'(x)E(x)$ and $Q(x)E'(x)$ are degree $n+2k-1$
and agree on $n+2k$ points

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Equation 2 implies 1:

$Q'(x)E(x)$ and $Q(x)E'(x)$ are degree $n+2k-1$
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$E(x)$ and $E'(x)$ have at most k zeros each.

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Can cross divide at n points.

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Can cross divide at n points.
$\implies \frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)}$ equal on n points.

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$\implies \frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)}$ equal on n points.
Both degree $\leq n$

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Can cross divide at n points.
$\implies \frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)}$ equal on n points.
Both degree $\leq n \implies$ Same polynomial!

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$E(x)$ and $E'(x)$ have at most k zeros each.
Can cross divide at n points.
$\Longrightarrow \frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)}$ equal on n points.
Both degree $\leq n \Longrightarrow$ Same polynomial! □

Last bit.

Fact: $Q'(x)E(x) = Q(x)E'(x)$ on $n+2k$ values of x.

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Proof: Construction implies that

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Proof: Construction implies that

$$Q(i) = R(i)E(i)$$
$$Q'(i) = R(i)E'(i)$$

Last bit.

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Proof: Construction implies that

$$Q(i) = R(i)E(i)$$
$$Q'(i) = R(i)E'(i)$$

for $i \in \{1,\dots n+2k\}$.

Last bit.

Fact: $Q'(x)E(x) = Q(x)E'(x)$ on $n+2k$ values of x.

Proof: Construction implies that

$$Q(i) = R(i)E(i)$$
$$Q'(i) = R(i)E'(i)$$

for $i \in \{1, \ldots n+2k\}$.

If $E(i) = 0$, then $Q(i) = 0$.

Last bit.

Fact: $Q'(x)E(x) = Q(x)E'(x)$ on $n+2k$ values of x.

Proof: Construction implies that

$$Q(i) = R(i)E(i)$$
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for $i \in \{1, \ldots n+2k\}$.

If $E(i) = 0$, then $Q(i) = 0$. If $E'(i) = 0$, then $Q'(i) = 0$.

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Proof: Construction implies that

$$Q(i) = R(i)E(i)$$
$$Q'(i) = R(i)E'(i)$$

for $i \in \{1, \ldots n+2k\}$.

If $E(i) = 0$, then $Q(i) = 0$. If $E'(i) = 0$, then $Q'(i) = 0$.
$\implies Q(i)E'(i) = Q'(i)E(i)$ holds when $E(i)$ or $E'(i)$ are zero.

Last bit.

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Proof: Construction implies that

$$Q(i) = R(i)E(i)$$
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If $E(i) = 0$, then $Q(i) = 0$. If $E'(i) = 0$, then $Q'(i) = 0$.
$\implies Q(i)E'(i) = Q'(i)E(i)$ holds when $E(i)$ or $E'(i)$ are zero.

When $E'(i)$ and $E(i)$ are not zero

Last bit.

Fact: $Q'(x)E(x) = Q(x)E'(x)$ on $n+2k$ values of x.

Proof: Construction implies that

$$Q(i) = R(i)E(i)$$
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Example: dealing with $\frac{x-2}{x-2}$ at $x = 2$.

Yaay!!

Berlekamp-Welsh algorithm decodes correctly when k errors!

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