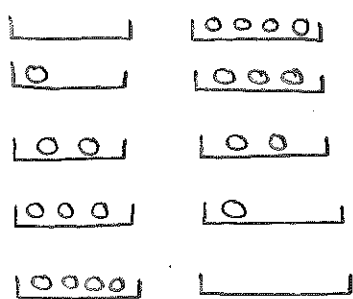# Note on #5 Finicky Bins   midterm 2 review problem

Consider a simpler problem.

Suppose there are 4 balls, 2 bins, and if ≥ 3 balls fall into a single bin then they will roll away.

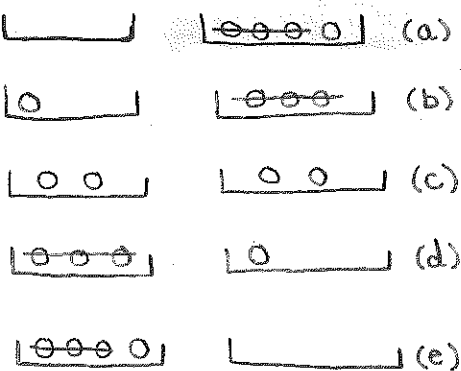How many different ways can 4 balls be placed across the 2 bins?

STEP 1

If no balls were to roll away, then there are 5 different ways.
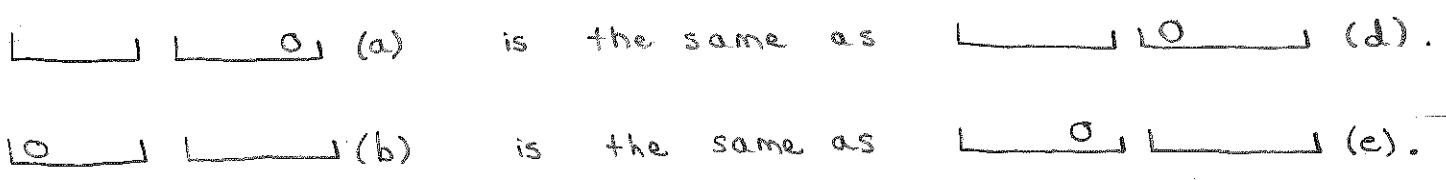
STEP 2
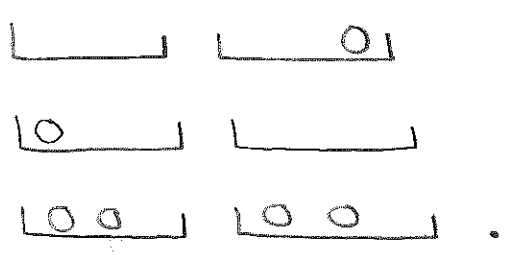
Now, let's cross out the balls that have rolled away.

(a)
(b)
(c)
(d)
(e)

In scenario (c), no balls are crossed out.

In the other 4 scenarios, some balls have been crossed out.

Some of the scenarios with crossed out balls are now the same.

⌊___⌋ ⌊__O⌋ (a)   is   the   same   as   ⌊_____⌋O⌊____⌋ (d).

⌊O___⌋ ⌊____⌋ (b)   is   the   same   as   ⌊__O__⌋⌊____⌋ (e).

In all, there are ⌈3 different ways⌉ the 4 balls can be arranged among the 2 bins. They are:

⌊____⌋ ⌊__O⌋

⌊O___⌋ ⌊____⌋

⌊OO⌋ ⌊O O⌋ .

Explaining the solution of simpler problem:

different ways, no balls crossed out ‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾

different ways, some balls crossed out

$$\binom{5}{1} \quad - \quad 2 \cdot \binom{2}{1} \quad + \quad \binom{2}{1}$$

↑
ways to place 4 balls among 2 bins if no balls were to roll away

↑
the 3 balls can be placed in one of 2 bins

↑
ways to place the remaining 1 ball among 2 bins

↑
There are $2\binom{2}{1} = 4$ scenarios in which balls will be lost. Once the balls fall out ½ of the scenarios will end up being the same.

2

Now consider the Finicky Bins problem as written.

Let's break down the solution.

$$\binom{10}{3} \qquad - \qquad 4 \cdot \binom{5}{3} \qquad = \qquad$$

$\overbrace{}^{\substack{\text{\# different ways,} \\ \text{no balls lost}}}$

ways, if no balls were to roll away $-$ # scenarios in which balls will be lost

↑
ways to place 7 balls among 4 bins if no balls were to roll away

↑
the 5 balls can be placed in one of 4 bins

↑
ways to place the remaining 2 balls among 4 bins

$$\frac{1}{4} \qquad \cdot \qquad \underbrace{4\binom{5}{3}} \qquad\qquad = \qquad \text{\# different ways, some balls lost}$$

↑
to account for duplicates once the balls fall out

↑
scenarios in which balls will be lost

All together,

$$\text{\#}\,\substack{\text{different} \\ \text{ways}} = \text{\#}\,\substack{\text{different} \\ \text{ways}}, \text{no balls lost} \quad + \quad \text{\#}\,\substack{\text{different} \\ \text{ways}}, \text{some balls lost}$$

$$= \binom{10}{3} - 4\binom{5}{3} + \binom{5}{3}.$$