1 Expectation and Variance

This problem will give you some practice calculating expectations and variances of random variables. Suppose that the random variable $X$ takes on 3 values, 10, 25, 70. Suppose $P[X = 10] = 0.5$, $P[X = 25] = 0.2$, and $P[X = 70] = 0.3$.

(a) What is $E[X]$?

(b) What is $E[X^2]$?

(c) What is $\text{var}(X)$?

2 Diversify Your Hand

You are dealt 13 cards from a standard 52 card deck. Let $X$ be the number of distinct values in your hand. For instance, the hand (A, A, A, 2, 3, 4, 4, 5, 7, 9, 10, J, J) has 9 distinct values.

(a) Calculate $E[X]$.

(b) Calculate $\text{Var}[X]$. 
3 Rolling Dice

(a) If we roll a fair 6-sided die, what is the expected number of times we have to roll before we roll a 6? What is the variance?

(b) Suppose we have two independent, fair \( n \)-sided dice labeled Die 1 and Die 2. If we roll the two dice until the value on Die 1 is smaller than the value on Die 2, what is the expected number of times that we roll? What is the variance?

(c) Let \( n = 6 \), so we are back to fair 6-sided die. Suppose we roll Die 1 until a 6 comes up, and we roll Die 2 until a 6 comes up. Let \( X \) be a random variable representing the number of times Die 1 is rolled before getting a 6, and let \( Y \) be the corresponding random variable for Die 2. Compute \( \mathbb{P}[\min(X, Y) = n] \) and \( \mathbb{P}[X + Y = n] \), where \( n \) is an integer.
4 Fishy Computations

Use the Poisson distribution to answer these questions:

(a) Suppose that on average, a fisherman catches 20 salmon per week. What is the probability that he will catch exactly 7 salmon this week?

(b) Suppose that on average, you go to Fisherman’s Wharf twice a year. What is the probability that you will go at most once in 2018?

(c) Suppose that in March, on average, there are 5.7 boats that sail in Laguna Beach per day. What is the probability there will be at least 3 boats sailing throughout the next two days in Laguna?