1 How Many Queens?

You shuffle a standard 52-card deck, before drawing the first three cards from the top of the pile. Let $X$ denote the number of queens you draw.

(a) What is $P(X = 0)$?

(b) What is $P(X = 1)$?

(c) What is $P(X = 2)$?

(d) What is $P(X = 3)$?

(e) Do the answers you computed in parts (a) through (d) add up to 1, as expected?

(f) Compute $E(X)$ from the definition of expectation.

(g) Suppose we define indicators $X_i$, $1 \leq i \leq 3$, where $X_i$ is the indicator variable that equals 1 if the $i$th card is a queen and 0 otherwise. Compute $E(X)$ using linearity of expectation.

(h) Are the $X_i$ indicators independent? Does this affect your solution to part (g)?

2 More Aces in a Deck

There are four aces in a deck. Suppose you shuffle the deck; define the random variables:

- $X_1$ = number of non-ace cards before the first ace
- $X_2$ = number of non-ace cards between the first and second ace
- $X_3$ = number of non-ace cards between the second and third ace
- $X_4$ = number of non-ace cards between the third and fourth ace
- $X_5$ = number of non-ace cards after the fourth ace
1. What is $X_1 + X_2 + X_3 + X_4 + X_5$?

2. Argue that the $X_i$ random variables all have the same distribution. Are they independent?

3. **Head Count**

Consider a coin with $P(\text{Heads}) = 2/5$. Suppose you flip the coin 20 times, and define $X$ to be the number of heads.

(a) What is the distribution of $X$?

(b) What is $P(X = 7)$?

(c) What is $P(X \geq 1)$?

(d) What is $P(12 \leq X \leq 14)$?