

1 Flippin' Coins

Suppose we have a biased coin, with outcomes H and T , with probability of heads $\mathbb{P}[H] = 3/4$ and probability of tails $\mathbb{P}[T] = 1/4$. Suppose we perform an experiment in which we toss the coin 3 times. An outcome of this experiment is (X_1, X_2, X_3) , where $X_i \in \{H, T\}$.

- (a) What is the *sample space* for our experiment?
- (b) Which of the following are examples of *events*? Select all that apply.
- $\{(H, H, T), (H, H), (T)\}$
 - $\{(T, H, H), (H, T, H), (H, H, T), (H, H, H)\}$
 - $\{(T, T, T)\}$
 - $\{(T, T, T), (H, H, H)\}$
 - $\{(T, H, T), (H, H, T)\}$
- (c) What is the complement of the event $\{(H, H, H), (H, H, T), (H, T, H), (H, T, T), (T, T, T)\}$?
- (d) Let A be the event that our outcome has 0 heads. Let B be the event that our outcome has exactly 2 heads. What is $A \cup B$?
- (e) What is the probability of the outcome (H, H, T) ?
- (f) What is the probability of the event that our outcome has exactly two heads?

2 Probability Warm-Up

- (a) Suppose that we have a bucket of 30 red balls and 70 blue balls. If we pick 20 balls out of the bucket, what is the probability of getting exactly k red balls (assuming $0 \leq k \leq 20$) if the sampling is done with replacement?
- (b) Same as part (a), but the sampling is without replacement.
- (c) If we roll a regular, 6-sided die 5 times. What is the probability that at least one value is observed more than once?

3 Probability Practice

- (a) If we put 5 math, 6 biology, 8 engineering, and 3 physics books on a bookshelf at random, what is the probability that all the math books are together?
- (b) A message source M of a digital communication system outputs a word of length 8 characters, with the characters drawn from the ternary alphabet $\{0, 1, 2\}$, and all such words are equally probable. What is the probability that M produces a word that looks like a byte (*i.e.*, no appearance of '2')?
- (c) If five numbers are selected at random from the set $\{1, 2, 3, \dots, 20\}$, what is the probability that their minimum is larger than 5? (A number can be chosen more than once, and the order in which you select the numbers matters)