

## 1 Roots

Recall that a polynomial of degree  $d$  has at most  $d$  roots. In this problem, assume we are working with polynomials over  $\mathbb{R}$ .

- (a) Suppose  $p(x)$  and  $q(x)$  are two different nonzero polynomials with degrees  $d_1$  and  $d_2$  respectively. What can you say about the number of solutions of  $p(x) = q(x)$ , in terms of  $d_1$  and  $d_2$ ? How about  $p(x) \cdot q(x) = 0$ ?
- (b) Consider the degree 2 polynomial  $f(x) = x^2 + ax + b$ . Show that if  $f$  has exactly one root, then  $a^2 = 4b$ .
- (c) What is the *minimum* number of real roots that a nonzero polynomial of degree  $d$  can have? How does the answer depend on  $d$ ?

## 2 Interpolate!

Find the lowest-degree polynomial  $P(x)$  that passes through the points  $(1, 4), (2, 3), (5, 0)$  modulo 7.

