1 Stable Marriage

Consider the set of men $M = \{1, 2, 3\}$ and the set of women $W = \{A, B, C\}$ with the following preferences.

<table>
<thead>
<tr>
<th>Men</th>
<th>Women</th>
<th>Women</th>
<th>Men</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A &gt; B &gt; C</td>
<td>A</td>
<td>2 &gt; 1 &gt; 3</td>
</tr>
<tr>
<td>2</td>
<td>B &gt; A &gt; C</td>
<td>B</td>
<td>1 &gt; 3 &gt; 2</td>
</tr>
<tr>
<td>3</td>
<td>A &gt; B &gt; C</td>
<td>C</td>
<td>1 &gt; 2 &gt; 3</td>
</tr>
</tbody>
</table>

Run the traditional propose-and-reject algorithm on this example. How many days does it take and what is the resulting pairing? (Show your work.)

2 Propose-and-Reject Proofs

Prove the following statements about the traditional propose-and-reject algorithm.

(a) In any execution of the algorithm, if a woman receives a proposal on day $i$, then she receives some proposal on every day thereafter until termination.

(b) In any execution of the algorithm, if a woman receives no proposal on day $i$, then she receives no proposal on any previous day $j$, $1 \leq j < i$. 
(c) In any execution of the algorithm, there is at least one woman who only receives a single proposal. (Hint: use the parts above!)

3 Be a Judge

For each of the following statements about the traditional stable marriage algorithm with men proposing, indicate whether the statement is True or False and justify your answer with a short 2-3 line explanation:

(a) There is a set of preferences for $n$ men and $n$ women for $n > 1$, such that in a stable marriage algorithm execution every man ends up with his least preferred woman.

(b) In a stable marriage instance, if man $M$ and woman $W$ each put each other at the top of their respective preference lists, then $M$ must be paired with $W$ in every stable pairing.

(c) In a stable marriage instance with at least two men and two women, if man $M$ and woman $W$ each put each other at the bottom of their respective preference lists, then $M$ cannot be paired with $W$ in any stable pairing.

(d) For every $n > 1$, there is a stable marriage instance with $n$ men and $n$ women which has an unstable pairing in which every unmatched man-woman pair is a rogue couple.