1 Stable Marriage

Consider the set of men $M = \{1, 2, 3\}$ and the set of women $W = \{A, B, C\}$ with the following preferences.

Men	Women
1	A > B > C
2	B > A > C
3	A > B > C

Women	Men
A	2 > 1 > 3
B	1 > 3 > 2
C	1 > 2 > 3

Run the traditional propose-and-reject algorithm on this example. How many days does it take and what is the resulting pairing? (Show your work.)

2 Propose-and-Reject Proofs

Prove the following statements about the traditional propose-and-reject algorithm.

(a) In any execution of the algorithm, if a woman receives a proposal on day i, then she receives some proposal on every day thereafter until termination.

(b) In any execution of the algorithm, if a woman receives no proposal on day i, then she receives no proposal on any previous day j, $1 \le j < i$.

(c) In any execution of the algorithm, there is at least one woman who only receives a single proposal. (Hint: use the parts above!)

3 Be a Judge

For each of the following statements about the traditional stable marriage algorithm with men proposing, indicate whether the statement is True or False and justify your answer with a short 2-3 line explanation:

(a) There is a set of preferences for n men and n women for $n > 1$, such that in a stable marriage algorithm execution every man ends up with his least preferred woman.

(b) In a stable marriage instance, if man M and woman W each put each other at the top of their respective preference lists, then M must be paired with W in every stable pairing.

(c) In a stable marriage instance with at least two men and two women, if man M and woman W each put each other at the bottom of their respective preference lists, then M cannot be paired with W in any stable pairing.

(d) For every $n > 1$, there is a stable marriage instance with n men and n women which has an unstable pairing in which every unmatched man-woman pair is a rogue couple.